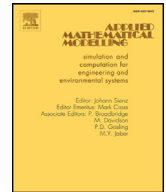




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## Applied Mathematical Modelling

journal homepage: [www.elsevier.com/locate/apm](http://www.elsevier.com/locate/apm)

## Global behavior of a host-parasitoid model under the constant refuge effect

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## ARTICLE INFO

## Article history:

Received 26 November 2013

Revised 25 January 2015

Accepted 3 September 2015

Available online xxx

## Keywords:

Host-parasitoid model

Steady-states

Boundedness

Local and global behavior

## ABSTRACT

In this paper, we discuss the qualitative behavior of a modified host-parasitoid model in which there is a constant number of hosts in a refuge, *i.e.*, the fixed number of hosts are safe from attack by parasitoid. More precisely, we study the boundedness and persistence, existence and uniqueness of positive equilibrium point, local asymptotic stability and global behavior of unique positive equilibrium point, and the rate of convergence of the solutions that converge to the unique positive equilibrium point of the modified host-parasitoid model. Some numerical examples are given to verify our theoretical results.

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## 1. Introduction

Typical insect species, host and parasitoid have a number of life-stages that include eggs, larvae, pupae and adults. In most cases eggs are attached to the outer surface of the host during its larval or pupal stage, or injected into the host's flesh. The larval parasitoids develop and grow at the expense of their host, consuming it and eventually killing it before they pupate. It is pointed out in [1] that a simple model for this system has the following set of assumptions:

- (i) Hosts that have been parasitized will give rise to the next generation of parasitoids.
- (ii) Hosts that have not been parasitized will give rise to their own progeny.
- (iii) The fraction of hosts that are parasitized depends on the rate of encounter of the two species; in general, this fraction may depend on the densities of one or both species.

Moreover, let  $H_n$  be the density of host species,  $P_n$  be the density of parasitoid species in generation  $n$ , respectively, and  $f(H_n, P_n)$  denotes the fraction of hosts not parasitized. Then our assumptions (i)–(iii) lead to:

$$\begin{aligned} H_{n+1} &= rH_n f(H_n, P_n), \\ P_{n+1} &= eH_n(1 - f(H_n, P_n)), \end{aligned}$$

where  $r$  is the number of eggs laid by a host that survive through the larvae, pupae and adult stages and  $e$  is number of eggs laid by a parasitoid on a single host that survive through larvae, pupae and adult stages. Assuming that the parasitoids search independently and their searching efficiency is constant  $a$ , leads to the Nicholson–Bailey model:

$$\begin{aligned} H_{n+1} &= rH_n \exp(-aP_n), \\ P_{n+1} &= eH_n(1 - \exp(-aP_n)). \end{aligned}$$

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Most natural parasitoid-host systems in nature are more stable than the Nicholson–Bailey seems to indicate and thus the model is not a satisfactory representation of real systems. There are a number of unrealistic assumptions in the Nicholson–Bailey model. For example, a constant reproductive rate of the host, a constant searching efficiency and a homogeneous environment are unrealistic assumptions. With additional realistic assumptions, the positive equilibrium can be locally asymptotically stable. In [2], the Nicholson–Bailey model is modified by taking

$$f(H_n, P_n) = \frac{EK}{H_n} + \left(1 - \frac{EK}{H_n}\right) \exp(-aP_n),$$

where  $K$  is said to be the carrying capacity of the environment for the population or the limiting value of the population that can be supported in a particular environment,  $E$  is the fraction of the host population  $E$ s carrying capacity that can be accommodated in safe refuges then  $EK$  has units of population density and represent the maximum of population density of the refuges.

Suppose environment is not homogeneous. Suppose the environment is patchy, so that a proportion of the host population may find a refuge and be safe attack by parasitoid. Arguing as in [2], we assume that there is a constant number of hosts  $N_0 = EK$  in a refuge. The hosts,  $N_0$ , are safe from attack by parasitoid. In this case, the modified Nicholson–Bailey model is given by:

$$\begin{aligned} H_{n+1} &= rN_0 + r(H_n - N_0) \exp(-aP_n), \\ P_{n+1} &= e(H_n - N_0)(1 - \exp(-aP_n)). \end{aligned} \quad (1)$$

Furthermore, if we take  $N_0 = 0$  then system (1) reduces to original Nicholson–Bailey model.

As it is pointed out in [3–6] the discrete time models governed by difference equations are more appropriate than the continuous ones when the populations are of non-overlapping generations. Particularly, the persistence, boundedness, local asymptotic stability, global character, and the existence of positive periodic solutions can be discussed more easily in case of difference equations as compared to differential equations. See [1] for introduction of mathematical models in biological sciences. Difference equations or discrete dynamical systems is diverse field which impact almost every branch of pure and applied mathematics. We refer [7–12] for basic theory and applications of nonlinear difference equations. It is very interesting to investigate the qualitative behavior of the discrete dynamical systems of non-linear difference equations. Particularly, to discuss the local asymptotic stability of equilibrium points and their global character. For more results for the systems of difference equations, we refer the reader to [13–23].

In [2], the numerical simulation results with various parameters showed that the steady-state of host and parasitoid population density coexist depend on the fraction of the host population's carrying capacity. In this paper, our aim is to study the boundedness and persistence, existence and uniqueness of positive equilibrium point, local asymptotic stability and global behavior of positive equilibrium point of the modified host-parasitoid system (1). Moreover, the rate of convergence of a solution that converges to the unique positive equilibrium point of system (1) is investigated.

## 2. Boundedness and persistence

**Lemma 1.** Let  $\{(H_n, P_n)\}$  be the solutions of system (1) with initial conditions  $H_0 > N_0 > 0$ ,  $P_0 > 0$  and  $r > 1$ . Then  $H_n > 0$ ,  $P_n > 0$  for all  $n > 0$ .

**Proof.** Assume that  $H_0 - N_0 > 0$ ,  $P_0 > 0$  and  $r > 1$ , then from first equation of system (1) we obtain  $H_1 - N_0 = rN_0 + r(H_0 - N_0) \exp(-aP_0) - N_0 = (r - 1)N_0 + r(H_0 - N_0) \exp(-aP_0) > 0$  and  $H_2 - N_0 = rN_0 + r(H_1 - N_0) \exp(-aP_1) - N_0 = (r - 1)N_0 + r(H_1 - N_0) \exp(-aP_1) > 0$ . For the sake of induction assume that  $H_{n-1} - N_0 > 0$ , then from first equation of system (1) it follows that

$$\begin{aligned} H_n - N_0 &= rN_0 + r(H_{n-1} - N_0) \exp(-aP_{n-1}) - N_0, \\ &= (r - 1)N_0 + r(H_{n-1} - N_0) \exp(-aP_{n-1}) > 0, \end{aligned} \quad (2)$$

for all  $n = 1, 2, \dots$ . Moreover, from (1) we have  $P_1 = e(H_0 - N_0)(1 - \exp(-aP_0)) > 0$ , and  $P_2 = e(H_1 - N_0)(1 - \exp(-aP_1)) > 0$ . Next, suppose that  $P_n > 0$ , then from second equation of system (1) and (2) it follows that

$$P_{n+1} = e(H_n - N_0)(1 - \exp(-aP_n)) > 0.$$

Finally, from first equation of system (1) it follows that

$$\begin{aligned} H_{n+1} &= rN_0 + r(H_n - N_0) \exp(-aP_n) \\ &= rN_0(1 - \exp(-aP_n)) + rH_n \exp(-aP_n) > 0, \end{aligned}$$

for all  $n = 0, 1, \dots$ .  $\square$

In the following theorem we will study the boundedness and persistence of system (1).

**Theorem 1.** Under the assumptions of Lemma 1, every positive solution  $\{(H_n, P_n)\}$  of system (1) is bounded and persists, if  $eaN_0(r - 1) > 1$  and  $rc < 1$ , where  $c = \exp\left(-\frac{a^2}{b-1}\right)$  and  $b = eaN_0(r - 1)$ .

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