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Analytical and numerical study of natural convection induced by a volumetric heat generation in inclined cavities asymmetrically cooled by heat fluxes

L. El Moutaouakil, Z. Zrikem*, A. Abdelbaki

Cadi Ayyad University, Faculty of Sciences Semlalia, LMFE (CNRST-URAC 27), BP 2390, Marrakech 40000, Morocco

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ABSTRACT

This paper reports an analytical solution, based on the parallel flow approximation, of the steady laminar natural convection in an inclined cavity filled with a volumetric heat generating fluid and cooled asymmetrically by imposed heat fluxes on the sidewalls. Numerical solutions of the full governing equations are used to determine the limits of validity of the analytical solution as a function of the controlling parameters (aspect ratio A, heat flux ratio r, inclination angle ϕ, Prandtl number Pr and Rayleigh number Ra). It is shown that the agreement between the analytical and numerical results is excellent for $15^\circ \leq \phi \leq 165^\circ$ if $A \geq 5$ and $Pr \geq 0.7$, regardless of the values of r and Ra (laminar regime). Analytical and/or Numerical predictions are also exploited to present the effects of the controlling parameters on the characteristics of the heat transfer and fluid flow within the cavity. Thus, in the middle of the cavity, it is found that the maximum temperature and its position depend strongly on the values of the controlling parameters.

1. Introduction

Natural convection in enclosures with volumetric heat generation occurs in many engineering systems related to electrical, chemical and nuclear industries [1–4]. For these systems, the knowledge of the heat transfer and fluid flow characteristics, as well as the maximum temperature reached, is necessary to optimize and control their operation.

In the literature, there are a large number of experimental, numerical and analytical studies that have treated the natural convection with internal heat generation in rectangular cavities of different aspect ratios A and inclinations ϕ. Among these works, one can mention those who have examined cavities filled with liquid metals (Prandtl number $Pr \ll 0.1$) whose generated volumetric heat is removed through certain walls of the enclosure that are maintained at a constant temperature [4–6]. Natural convection of volumetric heat generating fluids which are additionally heated by the cavity walls, kept at constant temperatures, has also been extensively studied for various fluids [2, 7–13]. Magnetohydrodynamic natural convection due to internal heating has also received a particular interest [14–16].

In this work, we focus on the two-dimensional laminar natural convection in an inclined rectangular cavity filled with a volumetric heat generating fluid having $Pr \geq 0.7$. The internal heat generated by the fluid is removed through the cavity walls. For this configuration, among the early works those of Kulacki et al. [1, 17–18] who have experimentally examined the natural convection induced by Joule heating of a horizontal fluid layer. The cooling of the fluid layer is done through the two isothermal

* Corresponding author. Tel.: +212 5 24 434649; fax: +212 5 24 4374 10.
E-mail address: zrikem@uca.ma (Z. Zrikem).

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Nomenclature

A	aspect ratio
C_T	dimensionless temperature gradient in y-direction
g	gravitational acceleration, m/s^2
H	height of the cavity, m
J	operator
k	thermal conductivity, W/mK
L	width of the cavity, m
p	pressure, Pa
P	dimensionless pressure
Pr	Prandtl number
q	heat flux, W/m^2
q'	volumetric heat flux, W/m^3
r	heat flux ratio
Ra	Rayleigh number
T	temperature, K
t	time, s
u, v	velocities, m/s
U, V	dimensionless velocities
x, y	Cartesian coordinates, m
X, Y	dimensionless Cartesian coordinates

Greek symbols

α	thermal diffusivity, m^2/s
β	thermal expansion coefficient, K^{-1}
θ	dimensionless temperature
ν	kinematic viscosity, m^2/s
ρ	density, Kg/m^3
τ	dimensionless time
ψ	stream function
ϕ	angle of inclination, °

Subscripts

1	wall 1
2	wall 2
c	conduction
h, 0	median $y = 0$
i	initial
l	limit
m	mean
max	maximum
min	minimum
v	y-direction

horizontal walls when the lower one is not considered adiabatic. Their investigations have allowed to develop correlations for estimating the heat transfer depending on the Rayleigh number Ra.

Lee and Goldstein [19], studied experimentally the laminar natural convection in a square cavity filled with salt water, generating a uniform volumetric heat. The cavity is cooled from its four walls maintained at the same constant temperature. The tilt angle ϕ of the cavity, measured from the horizontal, has been varied from 0° to 45° with a step of 15°. They found that the maximum temperature of the fluid is reached near the top corner of the cavity $\phi \geq 30°$ and $Ra \geq 5.10^4$. The same configuration was studied numerically by May [20], who found results in good agreement with those of Lee and Goldstein [19]. Unsteady natural convection of a heat generating fluid in a not inclined rectangular cavity with isothermal or adiabatic walls and of various aspect ratios has been studied numerically by Churbanov et al. [21]. For $A = 1$, the results obtained are in good agreement with those of May [20], including the outbreak of the oscillation of the convective flow.

Recently, a detailed parametric study was conducted numerically by Deshmukh et al. [22] on an inclined cavity ($0° \leq \phi \leq 45°$) filled with various volumetric heat generating fluids ($Pr = 0.71,\ 1\ and\ 7$) and of aspect ratios ranging from $A = 0.25$ to 6. The four walls of the cavity are maintained at the same cold temperature and the generated internal heat is increased gradually until an established unsteady flow is obtained. They validated their numerical code by comparing their results to those of Lee and Goldstein [19] and May [20]. It is shown that for low values of the Rayleigh number (low volumetric heat generation),

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