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Stability analysis of non-Newtonian rimming flow

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ABSTRACT

The rimming flow of a viscoelastic thin film inside a rotating horizontal cylinder is studied theoretically. Attention is given to the onset of non-Newtonian free-surface instability in creeping flow. This non-inertial instability has been observed in experiments, but current theoretical models of Newtonian fluids can neither describe its origin nor explain its onset. This study examines two models of non-Newtonian fluids to see if the experimentally observed instability can be predicted analytically. The non-Newtonian viscosity and elastic properties of the fluid are described by the Generalized Newtonian Fluid (GNF) and Second Order Viscoelastic Fluid (SOVF) constitutive models, respectively. With linear stability analysis, it is found that, analogously to the Newtonian fluid, rimming flow of viscous non-Newtonian fluids (modeled by GNF) is neutrally stable. However, the viscoelastic properties of the fluid (modeled by SOVF) are found to contribute to the flow destabilization. The instability is shown to increase as the cylinder rotation rate is lowered, from which the fluid accumulates in a pool on the rising wall. Viscoelastic effects coupled with this pooling cause the fluid's angular stretching, which is suggested to be responsible for this onset of instability.

1. Introduction

The problem of rimming flow has been investigated for many years because of its applications in industry. In most applications, a uniform, smooth film coating is desired [1]. However, experimentally, it was shown that rimming flow is characterized by wide variety of uneven, bulging steady-state film distributions and instabilities [2–7]. Depending on various physical parameters including cylinder rotation rate and cylinder filling fraction, this desirable smooth flow regime may or may not exist. Recently, Seiden and Thomas [8] and Seiden and Steinberg [9] experimentally studied a non-inertial instability unique to viscoelastic rimming flows. Even small concentrations of polymer resulted in free surface plume formation on the rising wall pools, giving rise to complex coagulation dynamics. It is of both practical and theoretical interest to describe such instability analytically.

Previous theoretical studies of rimming flow, however, have largely considered Newtonian fluids. Moffatt [10] first derived the value of the maximum amount of fluid a rotating cylinder can sustain. For masses above this critical value, gravitational forces overcome the cylinder's rotational drag and cause a fluid puddle to accumulate on the rising wall of the cylinder. In the lubrication approximation, O'Brien [11] showed that the position of the puddle on the cylinder wall can be represented by shock solutions. It was later shown that these shock solutions are stable [12–17]. However, these "pooling" solutions exhibit uneven bulges, and can be undesirable for coating applications.

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Rimming flow does exhibit smooth free surfaces for subcritical loads. The subcritical regime is characterized by small cylinder filling fraction and fast rotation rate. While rather smooth and uniform, the subcritical film has shown instability in experimental investigations. O'Brien [18] first considered the stability of the subcritical regime for Newtonian fluids. In a linear stability analysis, he showed that for this class of fluids the uniform subcritical solutions are neutrally stable. This linear stability analysis was extended in [19–24] by including higher order effects normally ignored in the lubrication approximation. Pressure differences at the top of the cylinder proved destabilizing, but adding surface tension stabilized the solution [25]. Inertia, however, demonstrated a significant destabilizing effect [23,26].

Although previous studies take many complicated effects into account, they do not consider non-Newtonian rheological effects when studying steady-state stability. Coating industries use polymers (e.g. such as polyethylene) that exhibit viscoelastic rheology that greatly deviates from a Newtonian behavior [27]. Polymers exhibit Newtonian rheology for small strains, but transition to shear-thinning for larger shear rates. They also exhibit much larger normal stresses than Newtonian fluids, so elongation and tension effects become significant [27]. To completely describe this important manufacturing process and the new experimentally detected instabilities seen at low Reynolds number [8,9], the effects of these non-Newtonian properties need to be characterized.

However, the non-Newtonian rimming flow, in general, and its stability, in particular, has not been extensively studied. Fomin et al. [28,29] proved that shear-thinning fluids described by the power-law, Ellis, and Carreau models lowered the maximal supportable load of the cylinder. Because shear-thinning inhibits the shear-force of the viscous drag of the cylinder, higher rotation rates are required to offset this gravitational-viscous imbalance. Through numerical investigation, Rajagopalan et al. [30] demonstrated that viscoelasticity raises the maximal supportable load.

In our study, the linear stability results obtained for Newtonian fluids are extended to non-Newtonian fluids. The effects of non-Newtonian shear thinning and of elastic normal stresses on the stability of subcritical steady-state rimming flow are studied with a restriction of small Deborah number. To solve the evolution equation governing time-dependent film thickness, the film's free surface is expanded as a normal mode perturbation of the steady-state and the resulting eigenvalue problem is solved. It is shown that, within the lubrication approximation, shear-thinning films are neutrally stable. It is further shown that viscoelasticity destabilizes the film's steady-state. A mechanism is proposed to explain the onset of the experimentally observed instability [8,9].

2. System model

Fig. 1 contains a schematic of rimming flow. A horizontal cylinder of radius r_0 is rotating in a counterclockwise direction θ with constant angular velocity Ω (discussion about a more realistic geometry with tilted cylinders can be found in [31]). A thin liquid film of thickness $h*(\theta, t)$ moves along the inner cylinder wall due to the gravity and the cylinder's rotational drag force. A cylindrical system of coordinates (r, θ, z) is used such that the z-axis coincides with the axis of the cylinder. The rest of the cylinder is modeled as being filled with rarefied gas of uniform pressure and negligible viscous traction at the liquid-gas interface. It is assumed the cylinder is sufficiently long such that the flow is two-dimensional. Three-dimensional effects have been discussed in some recent publications [32].

The governing equations for incompressible creeping flow are presented:

$$\nabla^* \cdot \boldsymbol{v}^* = 0, \tag{1}$$

$$\rho \boldsymbol{g} - \nabla^* p^* + \nabla^* \cdot \boldsymbol{\tau}^* = 0, \tag{2}$$

where vectors and tensors are denoted in boldface and dimensional variables with asterisks, $\nabla*$ is the gradient, $\boldsymbol{v}*$ is the fluid velocity vector with radial and angular components v_r^* and v_θ^*, $p*$ is the pressure, $\boldsymbol{g}$ is the gravity acceleration vector, and $\boldsymbol{\tau}*$ is the stress tensor deviator. Expressions and scaling laws for $\boldsymbol{\tau}*$ are specific to the constitutive model being used. Because two

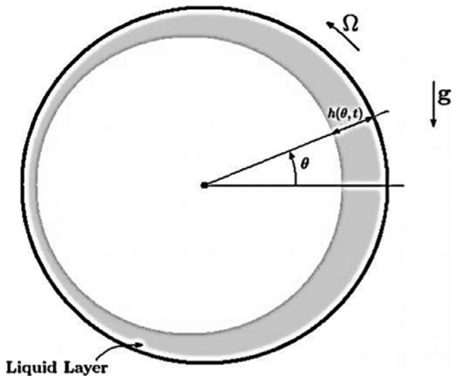

Fig. 1. A simple sketch of the rimming flow system (from [33]).