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Simultaneous recovery of the temperature and species concentration from integral equation model

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ABSTRACT

Absorption spectroscopy is an advanced tool for flow diagnostics in measuring multiple parameters of species. Such kinds of problems can be modeled by some integral equations with known kernel, aiming to the determination of the integrands from their integration values along all possible paths of injected lasers. This paper considers the parameters detection problems in combustion process, with the purpose of recovering the gas temperature and the concentration of burned gas simultaneously using injected lasers along two directions with multiple frequencies. After establishing the nonlinear integral equations describing the energy absorption process, this ill-posed model is transformed into a nonlinear optimization problem with some penalty terms. Then we present an alternative iteration scheme (AIS) to solve this problem. The convergence of the iterative sequence for AIS algorithm together with the estimate on the value of cost functional is established, ensuring that AIS can indeed generate a satisfactory approximate solution to the original optimization problem. Numerical implementations using simulant data are presented to show the validity of the proposed scheme.

1. Introduction

The conversion of energy is popular in the world, which provides fundamental energy supports on the realizations of many modern advanced techniques. Among all the energy conversion ways, combustion is one of the most widely used techniques. By combustion process, the energy conserved in gas or petroleum is transformed into the other forms such as motion energy, which have found many applications in modern traffics and spaceflight.

The monitoring and optimization for the combustion process are of great importance. First, the suitable temperature distributions for the burning gas are crucial to the safety of the combustion process and therefore have gotten much attention in engineering designs. Second, the concentration of the burned gas is of essential influence on the combustion process itself, especially for improving the combustion efficiency and reducing the pollutants such as nitrogen oxides. However, due to the extra high temperature in the combustion process, see engineering configuration in Fig. 1, the direct measurement for the parameters of burning gas is impossible. To overcome this difficulty, some instruments such as diode laser absorption sensors have been invented to catch some indirect measurement data related to the combustion process with the potential for the combustion control [1].

Once we have obtained these indirect data, the next step is to extract the concerned physical parameters such as the temperature of the burning gas from the measurement data, which is essentially to establish an appropriate mathematical model in

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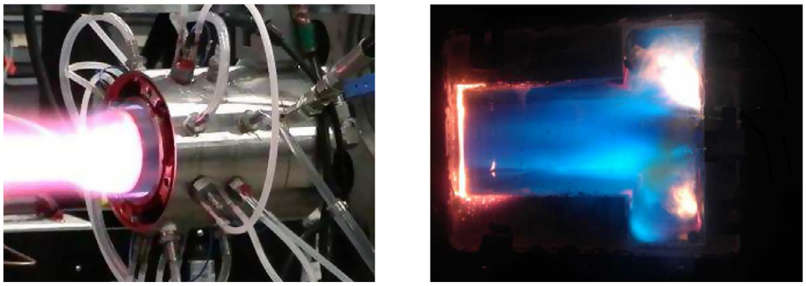

Fig. 1. The engineering configuration for the combustion (left) and its two-dimensional cross section (right).

terms of related physical law and then to construct an efficient numerical algorithm for getting these parameters, for example, see [2]. This step, aiming to the recovery of the system parameters from noisy measurement data related to the system indirectly, is called "parameter identification" in engineering societies, while the other terminology "inverse problem" is used in mathematical communities. The efficient algorithms for detecting system parameters are very hard to be established due to the following three reasons. First, the provided measurement data are in general not sufficient due to the technique restrictions, and consequently cannot determine the required parameters uniquely. Second, the unavoidable noise in the measurement may be highly amplified in the computational process due to the essences of the model, which causes the instability of the algorithms. Finally, most of the physical models concerning the parameters reconstruction are nonlinear. Therefore we need to establish some reconstruction algorithm for a *nonlinear ill-posed* problem which can recover the media parameters *approximately* and *stably*. The key technique for this purpose is the so-called regularizing strategy, which can be considered as a key part for the approximate well-posed mathematical model for the original ill-posed problem.

Although the absorption spectroscopy has been used widely in engineering areas for the detection of gas parameters for a long time, see [3–6], to the authors' knowledge, all the existing works are focusing on the numerical tests by general optimization schemes, for example, see [7–9], no attentions are paid to the special structure of the reconstruction model such as the semilinearity with respect to the species concentration, to say nothing of the mathematical analysis on the iteration schemes. In this paper, we establish an appropriate mathematical model for the following combustion process, aiming to the simultaneous reconstruction of the temperature and concentration of burned gas with the absorption spectroscopy data as our inversion input. The gas flow of high temperature moves with high velocity along some pipe which is assumed to be a cylinder, see Fig. 1. At the two-dimensional cross section of this cylinder, the lasers are injected with different frequencies along two perpendicular directions from one side and measure the remained energy at the opposite side. We try to recover the temperature and concentration distributions in the domain of gas burning from the measurement data. The energy absorption of laser injection has been applied for detecting the media structures in many applied areas, for example see [10] for detecting the tissue structure in medical imaging.

The above process using the energy absorption along injected laser lines can be described by an integral equation model for the parameters to be detected. However, the characteristic of line-of-sight (LOS) for this model makes this physical process hard to get higher spatial resolution for species parameters reconstructions. Moreover, the finite number of incident directions of laser paths makes the system parameters unable to be recovered uniquely. To overcome these uncertainty and instability for absorption spectroscopy techniques and the nonlinearity of the corresponding reconstruction problem, our strategy is to transform the integral equation model into an optimization problem with some penalty terms. Then we propose a semilinear alternative iteration scheme to solve this problem, which includes a linear iteration process for recovering the species concentration with fixed temperature and then updating the temperature by solving a nonlinear optimization problem with some penalty terms at each iteration step. Moreover, we propose a cutting-off strategy for ensuring the iterative solution in the admissible set, to guarantee the satisfactory approximation of our iterative solution to the sought parameters.

For general nonlinear ill-posed system

$$F_i(x) = y_i, \quad i = 1, \dots, m, \tag{1.1}$$

with some completely continuous operators $F_i\colon D_i \subset X \to Y$, there have been several well-developed schemes for solving (1.1) such as iterative-type regularization methods [11,12], Tikhonov-type regularization methods [13,14] or Landweber–Kaczmarz methods ([15–18]). The iterative-type and Tikhonov-type regularization methods for solving (1.1) can be unified in the framework of solving an operator equation

$$\mathcal{F}(x) = y \tag{1.2}$$

with $\mathcal{F} := (F_1, \dots, F_m)$ and input data $y = (y_1, \dots, y_m)$. The iterative-type schemes solve (1.2) by iteration process with the stopping step N as the regularizing parameter [12,13,19], while the Tikhonov-type schemes convert (1.2) into an optimization problem involving some penalty terms for the stability of the optimizer, with the weights of penalty terms as the regularizing parameters [14]. However, when the size of the system, i.e., the value of m, is very large, these two schemes require very huge computational memory, since they consider all the m-input conditions as a whole to yield approximate solution by iterations or optimizations. The Landweber–Kaczmarz method is essentially the iterative-type method using Landweber iteration [15,18].