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Production lot sizing with quality screening and rework

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ABSTRACT

Most production systems produce items which are of imperfect quality. Handling of the defective items varies by industry and product types. For example, defective items may be sold at discount in the apparel industry, or reworked in the automobile industry where the final product is very expensive. For simplicity, a common assumption in the literature is that items are lumped into two groups, non-defective and to-be-reworked products, ignoring the inspection time needed to identify the repairable items. Our paper explicitly integrates the inspection time into the economic production model with rework, and demonstrates the significant effect that the inspection time has on the results. We consider a manufacturing process with random supply and a screening process conducted during and at the end of production. We analyze two scenarios for dealing with the defective items produced: selling at a discount, and reworking. For each scenario, the demand during production is met using non-defective items only. The expected profit functions are developed using the renewal reward theory, and closed form expressions for the optimal production lot size are derived. Numerical analysis is performed to study the sensitivity of the expected profit and optimal lot size to various system parameters.

1. Introduction

We consider a machine producing a single item, with the possibility of producing a random proportion of defective items. To identify the defective items, screening is conducted at the end of the production period. Once identified, the defective items are reworked at a constant rate before they are returned to the inventory. A common assumption in the inventory literature with defectives is that the rework of a defective item is followed immediately after it is identified (Hayek and Salameh [1], Jamal et al. [2], Sarker et al. [3]). This assumption of continuous screening during production complicates the analysis and is not practical for most production systems, especially when the fraction of defective items is low and the production rate is high, which makes continuous screening during production very expensive. To simplify the analysis and the computation of the average inventory, it was assumed in the literature that the products are lumped into two groups, good products and to be reworked products. Our paper, on the other hand, avoids this assumption by integrating the screening process into the production lot sizing model with rework. During production, demand is met from non-defective items only. This implies that the system screens the items as they are being sold, i.e. the screening rate is the same as the demand rate before production ends. After production ends, a screening process is initiated for all the remaining products. In practice, decisions of when and where to place the screening stations are of special importance. The issues of comparing the screening strategies (after-production versus during production

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screening), and locating the screening stations, (whether the screening should be carried out after each production stage in a serial production line or at the end of the last stage), have been addressed in numerous research studies (Gurnani et al. [4]; Goyal et al. [5]; Tang [6]; Giri and Dohi [7]). Similarly to this paper, several research works in the literature have assumed screening at the end of production, for example, Ma, Gong, and Lin [8]. In this work, we analyze two models: In the first model, defective items identified at the end of the screening period are carried in inventory and sold at a discount as a single batch at the end of the production cycle. In the second model, defective items identified are reworked at a constant rate. We assume that no shortages are allowed.

For the model with rework, we assume that defective items are repairable, and no items are scrapped. This is true especially for expensive products which are not usually scrapped but reworked. Sarker et al. [3] discuss examples of such products, such as the metal book-shelves and defective filing cabinets. Another example is the wafer probe operation of wafer fabrication in the semiconductor manufacturing, where many types of deformations of the perfect integrated circuit can take place, as a result of the process disturbances (Lee [9]). Also, in automobile industries, the defective alignment of steering wheels is mostly repaired. Other examples include the air conditioning units, and ceiling fans components (Sarker et al. [3]). Scrapping such products would be very expensive, so companies collect defective items and repair them at a cost.

The models presented in this paper account for onsite and manual inspection as newly produced items are being consumed before the screening process begins. Obviously, applications for such models include systems where strict quality measures are enforced. Real world systems with strict quality control measures include the food industry. For example, a food safety law enforced in the United Kingdom (U.K.) resulted in retailers no longer being protected from liability by warranties or guarantees from suppliers, but they are required to make sure that the food they sell is of good quality (Bredahl et al. [10]). In such a case, the retailer inspects the newly received items at the same rate as they are being shelved for customer consumption before committing to a thorough inspection process of the entire production batch. Non-conforming food items cannot be reworked and have to be scrapped as represented by Model 1 or can be returned to the supplier which serves as a motivation for Model 2. On construction sites where the quality of material as well as on time delivery are critical factors, a project manager inspects the material to be consumed manually before accepting the order to identify damaged material (reports damaged items using check sheets) (Kerzner [11]). In such a case, material which is damaged is identified by the project manager at the same rate at which it is consumed where the damaged material can be sent back to the supplier production cycle for rework (Model 2).

Our models offer three practical aspects: (i) Screening starts right when production stops, due to a fast production process that makes screening during production difficult in practice, (ii) demand during production is met from non-defective items only, and (iii) once identified, defective items are carried in inventory and salvaged at the end of the production cycle, or reworked at a constant rate.

The paper is organized as follows: The literature is surveyed in Section 2. In Section 3, we present the mathematical formulation. First, the model with defective items sold at discount is presented in Section 3.1, and closed-form expression for the optimal lot size is obtained. Then the model with rework is analyzed in Section 3.2. Numerical and sensitivity analysis are conducted in Section 4 to investigate the effects of various model parameters on the expected profit and optimal lot size. A conclusion is provided in Section 5.

2. Literature review

Inventory models with imperfect quality items have received significant attention in the literature. For a survey on these models, we refer the reader to Yano and Lee [12]. One of the earliest work is Porteus [13] who incorporates the effect of defective items into the economic order quantity (EOQ) model, and analyzes the relationship between quality and lot size. Zhang and Gerchak [14] study a joint lot sizing and inspection policy for an EOQ model with defectives. Goyal and Cardenas-Barron [15] develop a practical approach for an economic production quantity (EPQ) model with defective items. Paknejad et al. [16] assume the number of defective items to be a random variable, and further assume stochastic demand and constant lead time. They show the optimality of a modified (s, Q) policy and obtain results for the case of exponential and uniform demand distributions. Salameh and Jaber [17] consider the realistic case where defective items are detected by a screening process and sold at discount at the end of the cycle. Quality control is obviously critical to any inventory system and has significant managerial implications which include customer satisfaction and demand among other implications. As a result, the impact of incorporating screening and quality control on the behavior of inventory systems is gaining interest in the recent literature. For this reason, the model presented in Salameh and Jaber [17] has received attention in the literature due to the wide applicability of screening and quality control in inventory systems. The authors in Khan et al. [18] extend the model of Salameh and Jaber [17] to include the case where defective items are either sold or replaced. The authors consider a model similar to Salameh and Jaber [17] where items salvaged as a single batch at the end of screening. The model accounts for Type 1 and Type 2 errors due to misclassification of items at the screening stage. Jaber et al. [19] extends Salameh and Jaber [17] by assuming that a shipment is coming from a distant supplier, so it is not possible to replace the imperfect items by placing a new order to the same supplier. Thus, they consider two models: The first assumes that imperfect items are sent to a repair shop and the second model assumes that imperfect items are replaced by good ones from a local supplier at a higher cost. Eroglu and Ozdemir [20] extend the model of Salameh and Jaber [17] to allow for shortage backordering. They assume the defective rate to be a random variable and classify defective items as scraps and imperfect. Liu and Cetinkaya [21] find the optimal lot sizes of a manufacturer and a buyer in an integrated supplier-buyer setting, with imperfect manufacturing process and quantity discounts. In their model, the supplier produces more than the buyer's order quantity to reduce the supplier's setup cost. They develop a model to determine the optimal lot sizes, retail