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Bifurcation analysis on delay-induced bursting in a shape memory alloy oscillator with time delay feedback

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ABSTRACT

The mechanism for the action of time delay in a non-autonomous system with two time scales is investigated in this paper. The original mathematical model under consideration is a shape memory alloy oscillator with external forcing. The delayed system is obtained by adding both linear and nonlinear time delayed position feedbacks to the original system. Typical bursting patterns can be presented, including symmetric fold/supHopf, double-fold/supHopf and supHopf/supHopf bursting when periodic forcing changes slowly. The time delay is taken as a variable parameter to investigate its effect on the dynamics of the system such as the stability and bifurcation. We calculate the conditions of fold bifurcation and Hopf bifurcation as well as its stability with the aid of the normal form theory and center manifold theorem. Through bifurcation analysis, we can identify that the occurrence and evolution of bursting dynamic depends on the magnitude of the delay itself and the strength of time delayed coupling in the model. Furthermore, we use phase space analysis to explore the associated mechanisms for the oscillator with multiple coexisting attractors. Numerical simulations are also included to illustrate the validity of our study.

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1. Introduction

In recent years, the applications based on shape memory alloys (SMA) which are metallic compounds with the ability to return to a previous shape or dimension, are receiving increasing attention. The remarkable properties of SMA are attracting much motivating different applications in several fields of sciences and engineering. The dynamical behavior of shape memory systems is addressed in different references [1–4]. The great number of shape memory applications in many fields of science motivates the research work on the nonlinear responses and bifurcations of shape memory oscillators. On the other hand, a rich class of solutions and bifurcations such as jump phenomena, pitchfork, period doubling, Hopf bifurcations, complete bubble structures culminating into chaos, can be presented by shape memory systems [5,6].

A typical dimensionless system of the SMA oscillator with external excitation, derived from thermo mechanical mode, can be expressed by

$$\ddot{x} + \alpha_1 \dot{x} + \alpha_2 x - \alpha_3 x^3 + \alpha_4 x^5 = k \cos(\theta t), \tag{1}$$

where $\alpha_{1,2,3,4}$ are positive parameters, $k > 0$ is the forcing amplitude, $\theta > 0$ is the forcing frequency. For the ordinary drive case, i.e., $\theta = O(1)$, some important dynamical behaviors such as the primary resonance, the secondary resonance and the free

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resonance of the system, are studied by Piccirillo et al. [7,8]. In recent work, some researchers find especially when the external excitation frequency is far less than the natural frequency, i.e., $0 < \theta \ll 1$, some interesting hysteresis cycles can be easily obtained by a slowly changing external forcing, passing through the appointed bifurcation values of the unforced system periodically [9–11]. It is obviously that system (1) has more than two coexisting attractors, which are associated with different parameter conditions. Such a system will be considered in this paper. Dynamics in this case can behave in periodic oscillation with large-amplitude oscillations characterized by small amplitude oscillations which is called the bursting phenomena [12–16].

Up to now, the analysis and classification of bursting mechanism are mainly devoted to systems without any delay. However, delay phenomenon is frequently encountered in many fields of science and engineering [17,18]. Time delayed feedback control has become an important tool for describing and controlling various nonlinear phenomena in numerous and diverse fields [19–22]. One of the simplest but widely used time delayed systems is obtained by adding a time delayed position feedback to a system [23–26]. Thus, we add a time delay (linear or nonlinear) position feedback to Eq. (1), one can obtain the following new closed-loop delayed system

$$\ddot{x} + \alpha_1\dot{x} + \alpha_2 x - \alpha_3 x^3 + \alpha_4 x^5 = k\cos(\theta t) + A_1 x(t-\tau) + A_2 x^3(t-\tau), \tag{2}$$

where τ is a time delay, $A_{1,2}$ means gain coefficient about the delay position feedback. If $A_{1,2} > 0$, it means a positive position feedback, and negative feedback if $A_{1,2} < 0$. For the external frequency deviates far from the frequency of self-excited vibration in the shape memory alloy oscillator, i.e., two time scales evolve in the vector field, bursting oscillations may occur. We hope to use the time delay to control different bursting oscillations like a “switch” which either creates or regulates bursting motions by varying the values of the time delay. Only the results for position feedback are presented in this paper. The velocity feedback will be discussed in a different paper.

The rest of this paper is organized as follows. In Section 2, an analysis of the bifurcations and dynamics in uncontrolled system related to Eq. (1) is obtained. The stability analysis and the conditions of Hopf bifurcation including its direction related to Eq. (2) are studied in Section 3. The dynamics are obtained as a function of the delayed position feedback and the external periodic forcing. In Section 4, bursting mechanism of symmetric fold/supHopf, double-fold/supHopf and supHopf/supHopf type under different parameter conditions is discussed. Finally, Section 5 concludes the paper.

2. Bifurcations and hysteresis cycles in uncontrolled system

Our study is started from the uncontrolled system in the absence of time delay while forced excitation changes slowly. Converting system (1) into an autonomous form by considering $k\cos(\theta t)$ as a control parameter of δ, we obtain the system

$$\begin{cases} \dot{x} = y \\ \dot{y} = -\alpha_1 y - \alpha_2 x + \alpha_3 x^3 - \alpha_4 x^5 + \delta. \end{cases} \tag{3}$$

Denoting the equilibrium points as (x_0, y_0), obviously $y_0 = 0$, while x_0 is decided by the algebraic equation:

$$-\alpha_2 x + \alpha_3 x^3 - \alpha_4 x^5 + \delta = 0 \tag{4}$$

The numbers and bifurcation behaviors of these equilibrium points are determined by the values of the parameters of (4). For the fixed parameters $\alpha_1 = 0.1$, $\alpha_3 = 2$ and $\alpha_4 = 2$, equilibrium-point curve on double-parameter bifurcation set (δ, α_2) related to Eq. (3) is computed and plotted in Fig. 1, where $CP_1 = (0.46, 1.41, 0.61)$, $CP_3 = (-0.46, 1.41, -0.61)$ are supercritical cusp bifurcation points and $CP_2 = (0, 0, 0)$ is subcritical cusp bifurcation point [27].

With proper slow forcing, oscillations in Eq. 1 may exhibit hysteresis cycles, which are bursting patterns of point–point type with two or four jumps. Such dynamics can be easily controlled by modulating the bifurcation parameters. The associated dynamical mechanisms of the closed cycles with two or four jumps can be interpreted by parameters bifurcation behaviors related to Eq. (3). For example, fix $\alpha_1 = 0.1$, $\alpha_3 = 2$, $\alpha_4 = 2$, $k = 3$, $\theta = 0.01$, and in case of $\alpha_2 = 0.6$, a hysteresis cycle with two jumps is obtained and shown in Fig. 2(a). Oscillations switch between the two different attractors by fold bifurcations that form such hysteresis cycle with two jumps. Further increase of the parameter of α_2, may lead to transition of the trajectories with two jumps and then there exists a hysteresis cycle with four jumps. In case of $\alpha_2 = 0.7$, Fig. 2(b) shows the hysteresis cycle jumping between the attractor on the side and the middle attractor twice. Oscillations switch between the side and the middle attractors by fold bifurcations twice, which exhibit a transition in two coexisting attractors. Hysteresis cycles with two or four jumps can be distinguished by the attraction domain corresponding to different coexisting attractors.

3. Bifurcations and stability analysis in controlled system

In this section, the bifurcation behavior of the delayed oscillator of (2) is studied. Similarly, considering $k\cos(\theta t)$ as a control parameter of δ, system (2) can be converted into (5),

$$\begin{cases} \dot{x} = y \\ \dot{y} = -\alpha_1 y - \alpha_2 x + \alpha_3 x^3 - \alpha_4 x^5 + \delta + A_1 x_\tau + A_2 x_\tau^3, \end{cases} \tag{5}$$

where $x_\tau = x(t-\tau)$. Denoting the equilibrium points as (x_0, y_0), obviously $y_0 = 0$, while x_0 is decided by the algebraic equation

$$-\alpha_2 x + \alpha_3 x^3 - \alpha_4 x^5 + A_1 x + A_2 x^3 + \delta = 0, \quad \text{i.e.} \quad -(\alpha_2 - A_1)x + (\alpha_3 + A_2)x^3 - \alpha_4 x^5 + \delta = 0. \tag{6}$$

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