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Higher order compact scheme for laminar natural convective heat transfer from a sphere

T.V.S. Sekhar [a,*], B. Hema Sundar Raju [b], P.V.S.N. Murthy [c]

[a] School of Basic Sciences, Indian Institute of Technology, Bhubaneswar 751007, India
[b] Department of Mathematics, National Institute of Technology, Silchar 788010, India
[c] Department of Mathematics, Indian Institute of Technology, Kharagpur 721302, India

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ABSTRACT

Steady laminar natural-convective heat transfer from a solid sphere is studied by solving the complete Navier–Stokes and energy equations using higher order compact scheme in spherical polar coordinates. Results are obtained for Grashof numbers in the range 0.05 –125 and for Prandtl numbers ranging from 0.72 to 7. The local Nusselt number and mean Nusselt number are calculated and compared with available experimental and theoretical results. Streamlines, vorticity lines and isotherms are plotted. It is found that the scheme captures *mushroom shaped front* in temperature contours which has been observed experimentally in the literature. The same phenomenon is not captured by upwind method.

1. Introduction

Natural convection from cylinders and spheres have been studied for many years because of their fundamental importance in industrial applications such as boilers, digesters, furnaces, etc. The natural convective heat transfer from a horizontal cylinder is analyzed under various surface boundary conditions using different mathematical techniques [1–3]. The authors in [4–8] studied experimentally the natural convective heat transfer from a sphere. The full Navier–Stokes (N–S) equations for natural convective heat transfer over a sphere was first solved numerically by Geoola and Cornish [9]. It was further extended by the authors for unsteady case [10] and they used first order upwind differences to convective terms. Farouk [11] solved complete steady state N–S equations for wide range of Rayleigh numbers. The transient laminar free convection around an isothermal sphere is solved numerically by Fujii et al. [12] and Riley [13]. Jai and Gogos [14] solved the complete N–S equations for natural convective heat transfer from isothermal sphere by employing finite volume method. Higher order compact schemes (HOCS) are invariably applied for N–S equations in cartesian coordinates [15–18] and are applied less to flow problems in curvilinear coordinate systems [19–21]. An attempt for improved accuracy is achieved by Saitoh et al. [22], in their study of natural convection heat transfer from horizontal circular cylinder by employing traditional fourth order central differences. The forced convection heat transfer from a circular cylinder is analyzed by Baranyi [23] using traditional fourth order central differences for diffusion terms and pressure derivatives while modified third order upwind scheme is employed for convective terms. Jiten et al. [24] developed fully compact higher order computation of steady-state natural convection in a square cavity. Sekhar and Raju [25] developed an higher order compact scheme to solve complete Navier–Stokes and energy equations for laminar natural convective heat transfer from

* Corresponding author. Tel.: +91 6742576077.
E-mail address: sekhartvs@iitbbs.ac.in, sekhartvs@yahoo.co.in (T.V.S. Sekhar).

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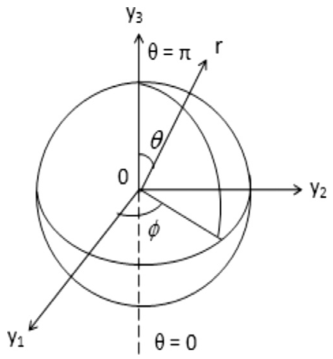

Fig. 1. Spherical polar coordinates.

horizontal circular cylinder. Sekhar and Raju [26] developed an efficient higher order compact scheme to capture forced convective heat transfer solutions from a sphere in spherical geometry.

The present paper is concerned with solving the complete Navier–Stokes and energy equations using higher order compact scheme for the laminar natural-convective heat transfer from a solid sphere in spherical polar coordinates.

2. Basic equations

The dimensionless equations for steady, laminar natural convection flow can be written in spherical polar coordinates as shown in Fig. 1 by applying the transformation $r = e^{\xi}$ using the Boussinesq approximation as follows [9,10] :

The velocity components:

$$q_r = -\frac{e^{-2\xi}}{\sin\theta}\frac{\partial\psi}{\partial\theta}, \qquad q_\theta = \frac{e^{-2\xi}}{\sin\theta}\frac{\partial\psi}{\partial\xi}. \tag{1}$$

The stream function equation:

$$\frac{\partial^2\psi}{\partial\xi^2} - \frac{\partial\psi}{\partial\xi} + \frac{\partial^2\psi}{\partial\theta^2} - \cot\theta\frac{\partial\psi}{\partial\theta} = e^{3\xi}\sin\theta\omega. \tag{2}$$

The vorticity transport equation:

$$\frac{\partial^2\omega}{\partial\xi^2} + \frac{\partial\omega}{\partial\xi} + \cot\theta\frac{\partial\omega}{\partial\theta} + \frac{\partial^2\omega}{\partial\theta^2} - \omega\csc^2\theta + e^{\xi}Gr\left(\frac{\partial T}{\partial\xi}\sin\theta + \frac{\partial T}{\partial\theta}\cos\theta\right)$$
$$= \frac{1}{e^{\xi}\sin\theta}\left(\frac{\partial\psi}{\partial\xi}\frac{\partial\omega}{\partial\theta} - \frac{\partial\psi}{\partial\theta}\frac{\partial\omega}{\partial\xi} - \frac{\partial\psi}{\partial\xi}\omega\cot\theta + \omega\frac{\partial\psi}{\partial\theta}\right). \tag{3}$$

The energy equation:

$$\frac{\partial^2 T}{\partial\xi^2} + \frac{\partial T}{\partial\xi} + \cot\theta\frac{\partial T}{\partial\theta} + \frac{\partial^2 T}{\partial\theta^2} = \frac{Pr}{e^{\xi}\sin\theta}\left(\frac{\partial\psi}{\partial\xi}\frac{\partial T}{\partial\theta} - \frac{\partial\psi}{\partial\theta}\frac{\partial T}{\partial\xi}\right). \tag{4}$$

where Gr is the Grashof number based on the radius of the sphere defined as follows:

$$Gr = \frac{R^3\beta g(T_S - T_\infty)}{\nu^2},$$

where R is the radius of the sphere, g is the gravitational acceleration, β is the thermal coefficient of volumetric expansion, ν is the kinematic viscosity, T_S is the temperature of the sphere surface and T_∞ is the temperature of the ambient flow. T is the non-dimensionalized temperature, defined by subtracting the ambient flow temperature T_∞ from the temperature and dividing by $T_S - T_\infty$ and Pr is the Prandtl number defined as the ratio between kinematic viscosity (ν) and thermal diffusivity (α). Eqs. (1)–(4) are dependent upon the assumption that the only body force operating is that of gravity and that the temperature variations within the fluid are not large, so that Boussinesq's approximation can be applied and the density to be treated as a constant in all terms of the transport equations execept the buoyancy term. Other fluid properties such as the viscosity, specific heat and thermal conductivity are taken to be constant. Eqs. (1)–(4) are subject to the following boundary conditions.

$$\text{On the surface of the sphere } (\xi = 0): \psi = \frac{\partial\psi}{\partial\xi} = 0, \omega = \frac{1}{\sin\theta}\frac{\partial^2\psi}{\partial\xi^2}, T = 1$$
$$\text{At large distances from the sphere } (\xi \to \infty): \psi = \omega = T = 0$$
$$\text{Along the axis of symmetry } (\theta = 0 \text{ and } \theta = \pi): \psi = 0, \omega = 0, \frac{\partial T}{\partial\theta} = 0.$$

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