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Coexistence states of a periodic cooperative reaction-diffusion system with nonlinear functional response☆

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ABSTRACT

In this paper, by using the existence and comparison conclusions for the *T*-periodic quasi-monotone nondecreasing system, we investigate a cooperative system with fractional functional response and the homogeneous Dirichlet boundary conditions. We mainly discuss the existence, uniqueness and nonexistence problems of positive solutions of the system by the method of upper and lower solutions. The numerical examples show that the relevant results hold with certain parameter values.

1. Introduction

In recent years attention has been given to periodic behavior of solutions of parabolic boundary-value problems arising from biological, chemical and physical systems. Different methods for the existence problems have been used, and most of the discussions on coupled systems are for special model problems such as the Lotka–Volterra model. In this paper, we consider the following periodic cooperative system with diffusion

$$\begin{cases} u_t - \Delta u = u\left(a - u + \dfrac{cv}{\gamma + v}\right), & t > 0,\ x \in \Omega, \\ v_t - \Delta v = v(b - v + du), & t > 0,\ x \in \Omega, \\ u(t,x) = 0, \quad v(t,x) = 0, & t > 0,\ x \in \partial\Omega, \\ u(t,x) = u(t+T,x), \quad v(t,x) = v(t+T,x), & t \geq 0,\ x \in \Omega, \end{cases} \tag{1.1}$$

where Ω is a bounded domain in R^N with smooth boundary $\partial\Omega$, u and v represent the densities of two species. The coefficients a, b, c, d, are all assumed to be positive T-periodic and smooth functions on $[0,\infty) \times \overline{\Omega}$, T is a fixed positive constant. a and b are the growth rates of u and v; c, d describe the interactions between the species. The interaction term for one species is known as saturating interaction, where γ measures the saturation level of the species and is assumed to be positive constant. Model (1.1) is more realistic than usual cooperation model for some kinds of species. This system describes the situation whenever the densities of the two species are large enough, one of the species displays the saturation at first, and the rate at which the

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species increase is below some maximum value no matter how large the other species might be (for more detailed discussions and biological background, see [1,2] for example).

The cooperation systems of time-independent and time-dependent have been widely studied in recent years, and most of the discussions were devoted to the classical cooperation systems [3–9]. For example, in [3], by discussing the properties of a linear cooperative system with homogeneous Neumann boundary condition, a few necessary and sufficient conditions for the existence of positive solutions of an elliptic cooperative system in terms of the principal eigenvalue of the associated linear system were established, and some local stability results for the positive solutions were also obtained. In [4–8], some cooperative systems with two or multi-species were studied. By spectral analysis method, monotone iterative technique, maximum principle, regularity theory and fixed point index theory, etc., the authors discussed the existence, nonexistence, blow-up and some other related problems of positive solutions of these systems. Especially, the blow-up estimates for solutions of a parabolic system arising in a cooperating three-species food chain model were investigated in [4,5], the existence of global solutions and blowup solutions were given in [4], and some upper and lower bounds of blow-up rate were obtained in [5]. In [6], for a quasi-monotone increasing system, the authors were concerned with the coexistence problem of the Volterra–Lotka model of two cooperating species, they gave a rather detailed analysis for the steady-state solutions. A two predator mutualists which cooperate in hunting for one prey in two patches was studied in [7]. The author assumed that both predators were able to sustain higher population numbers due to their mutualism, and the cooperation between the two predators was stated clearly. Numerical studies showed that the system would undergo a Turing bifurcation at a critical value of the bifurcation parameter. For a classical semi-linear elliptic cooperative system [8], the author established some general results on the existence of positive solutions to the system by virtue of some a priori estimates.

For the time-periodic models, many problems were investigated and lots of valuable results were established. However, most of the existing works were devoted to the cases of single species systems, competition or predator-prey systems, such as [10,11] for single species systems, [12–14] for competition systems, [15–17] for predator-prey systems. Sufficient conditions for the existence of periodic positive solution to these different systems were discussed in various extents. In [18,19], the T-periodic cooperative models were concerned. In [18], using the existence and comparison conclusions for the T-periodic quasi-monotone nondecreasing systems given by Pao [20] and iterative method, the authors investigated the T-periodic classical Lotka–Volterra two species cooperating model and gave some existence, uniqueness and nonexistence results. Some parallel results of the corresponding stationary system were also given. Motivated by Wang and Li [18], in [19], the authors discussed the T-periodic three-species cooperating model and established some sufficient conditions for the existence and estimates of coexistence states. Moreover, a few sufficient or necessary results for the existence of positive steady state of the model were also presented. Compared with [18], the existence problems of positive solutions for three-species model are more difficult than that of two-species model, since the constructions of the upper and lower solutions with different conditions are more complicated.

System (1.1) with constant coefficients has been studied. For instance, some results on the estimates and existence of positive solutions for the system were given in [21], where the interest was to search for the coexistence states for the corresponding elliptic system. The techniques involved in the arguments relied on the priori estimates, the global bifurcation theory, the fixed point index theory and the linear stability theory. In [22], the author investigated the asymptotic behavior of the time-dependent solutions of System (1.1) with constant coefficients and homogeneous Neumann boundary conditions. Some conditions were given to the rate constants so that for every initial function the corresponding time-dependent solution would converge to one of the nonnegative constant steady-state solutions as time tended to infinity. The convergence result led to the existence and uniqueness of a steady-state solution and the global asymptotic stability of a given nonnegative constant steady-state solution. Furthermore, some coexistence, permanence and extinction results for the system in terms of ecological dynamics were also obtained by considering the convergence property. For detailed information on ecological dynamics of more systems, one may refer to [23,24].

If the coefficients are assumed to be time-periodic functions on $[0, \infty) \times \overline{\Omega}$, it is possible to seek for the time-periodic positive solutions for (1.1). The periodic solutions are not only of theoretical interest but also have many practical applications. For example, optimal management of renewable resources has become an increasingly interesting topic in the recent decades, the exploitation of biological resources and the harvesting of population species are commonly practiced, such as in fishery, forestry and wildlife management. If the management of artificially control is carried out reasonably and properly, then the level of seasonal periodic variation might occur, this would be reflected in the system parameters which represent some real species. Moreover, if the harvesting is infrequent and periodic, the resources can be used sustainably. This indeed demands reasonable control of the harvesting yield in order to form a good ecological environment and avoid the occurrence of resource exhaustion. Biological speaking, if the populations exhibit periodicity, they are prone to persist for a long time, which are beneficial for protecting the diversity of species. Of particular interests from the point of view of applications would be such population models in which the birth rates, rates of interaction, and environmental carrying capacities are periodic on a seasonal scale. In this paper, we consider System (1.1) by assuming the coefficients a, b, c, d of T-periodic functions with respect to the time variable t, in order to allow the phenomenon of seasonal periodic variations or day-night cycles occur. We establish the existence, uniqueness and nonexistence results mainly by using the comparison techniques, that is, by the methods of upper and lower solutions and its associated monotone iterations. Here the similar arguments as in [18,19] are employed and extended to prove the results of problem (1.1) for the case of nonlinear functional response. Since the functional response in (1.1) is nonlinear and different from the classical cooperation models, it is difficult and complicated in constructing the upper and lower solutions for the corresponding problems. We think that the whole analysis implies that there are certain similarities between periodic and steady-state systems, and our results are applied to three or more species or substance model problems arising