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Observer design for one-sided Lipschitz descriptor systems

Ali Zulfiqar, Muhammad Rehan*, Muhammad Abid

Department of Electrical Engineering, Pakistan Institute of Engineering and Applied Sciences (PIEAS), P.O. Box 45650, Islamabad, Pakistan

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A B S T R A C T

This paper describes the design of a nonlinear observer for a class of nonlinear descriptor systems with one-sided Lipschitz nonlinearities. To provide a general framework applicable to a large class of systems, nonlinearities and disturbances are considered at the state as well as at the output equations. Non-strict and strict bilinear matrix inequality (BMI)-based robust observer synthesis schemes are provided by utilizing the one-sided Lipschitz condition, the concept of quadratic inner boundedness, the generalized Lyapunov theory for singular systems, and the L_2 gain reduction. The BMI-based condition is converted into the linear matrix inequality (LMI)-based condition by utilizing change of variables for straightforward computation of the observer gain matrix. In contrast to the traditional observer schemes for one-sided Lipschitz nonlinear systems, the resultant scheme is applicable to singular systems, capable of dealing with nonlinearities at the output equation and appropriate for treating with disturbances. Two simulation examples for the one-sided Lipschitz and the singular one-sided Lipschitz nonlinear systems are provided to validate the proposed observer synthesis methodologies.

1. Introduction

Descriptor systems, composed of both the ordinary differential equations (ODEs) and the algebraic equations, appearing from a more precise inherent modeling methodology, representing behavior of a more general class of systems (than the traditional state-space models), have emerged in many scientific and engineering disciplines such as biological systems, chemical processes, physical systems, electrical circuits, power systems and mechanical structures in addition to the social and economic fields [1–9]. Descriptor systems are also called singular systems, for which the initial condition always satisfies some algebraic constraints in contrast to the traditional ODE-based models. Observer design for these descriptor systems to estimate their state vectors has vast applications in controller synthesis, filtering, and fault diagnosis [2,3,6,10–13]; however, extra efforts are required to apply the generalized Lyapunov theory based on LaSalle's principle to ensure convergence of the estimation error. Considerable research activities concerning observer design for linear (see for example, [1–3,10,12,14]) as well as nonlinear (see, for example, [3,6,11,13]) descriptor systems have been carried out in the past decade.

In the recent years, one-sided Lipschitz nonlinear systems have become an important class of nonlinear systems due to their ability in representing a more general form compared to the Lipschitz systems and owing to superiority of the one-sided Lipschitz constant than the Lipschitz constant. Thanks to their numerous applications, the observer design problem has been studied in the works [15–19] for the one-sided Lipschitz systems. In [15], conditions for existence of an observer and a preliminary methodology for the observer gain selection are presented. The concept of quadratic inner boundedness was introduced in [16], which was

* Corresponding author. Tel.: +92 51 2207381x3443; fax: +92 51 2208070.
E-mail addresses: alizulfiqar10@gmail.com (A. Zulfiqar), rehanqau@gmail.com (M. Rehan), mabid@pieas.edu.pk (M. Abid).

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found to be interesting for linear matrix inequality (LMI)-based and Riccati equation based full-order and reduced-order observer synthesis [17,18]. Further, a discrete-time observer design approach, providing Lyapunov-based stability formulizations, has been presented in the recent work [19,20]. Note that these observer design schemes cannot be applied to the descriptor systems, containing both ordinary differential and algebraic equations. Further, the approaches [15–19] assume presence of nonlinear dynamics in the state equation only; though, incorporation of a nonlinear function into the output equation provides a more general description of a system and, hence, a less conservative observer design framework. Moreover, the effects of perturbations and disturbances, to ensure robust state estimation, have not been focused in these studies.

In this paper, a novel robust observer design methodology for the one-sided Lipschitz descriptor systems in the presence of both the input and the output nonlinearities in addition to disturbances is considered. By utilizing the one-sided Lipschitz condition, the quadratic inner boundedness, the generalized Lyapunov stability theory and the L_2 gain reduction, first a non-strict bilinear matrix inequality (BMI)-based condition is developed, which is further transformed into strict BMIs using a proper change of variables. Furthermore, an LMI-based formulation, suitable for computing the observer gain matrix, is provided herein by utilizing an exceptional change of variables to ensure asymptotic convergence of the state estimation error to origin and to accomplish robustness against L_2 norm bounded disturbances. The main contribution of the paper is summarized below:

(1) To the best of authors' knowledge, the observer design problem for the one-sided Lipschitz descriptor systems is addressed for the first time.
(2) The present work provides a less conservative observer design framework for the one-sided Lipschitz nonlinear systems by considering nonlinearities in the output as well as the state equations.
(3) Robustness against perturbations is addressed by considering disturbance at the state equation as well as at the output equation in contrast to the works [15–20].

Simulation results of the observer design methodologies for the one-sided Lipschitz and the singular one-sided Lipschitz nonlinear systems are provided to verify effectiveness of the proposed schemes.

This paper is organized as follows. System description is provided in Section 2. Section 3 presents observer design schemes utilizing BMIs and LMIs. Simulation results for the state estimation are detailed in Section 4. Section 5 draws conclusions of the study.

Standard notation is used throughout the paper. The matrix inequality $Z > 0$ (or $Z \geq 0$) for a symmetric matrix Z implies that Z is a positive-definite (or semi-positive-definite) matrix. Transpose of a matrix Z is represented by Z^T. $\langle a, b\rangle$ represents the inner product of two vectors a and b in R^n space. The L_2 gain of a nonlinear system with input and output vectors w and z (of appropriate dimensions) under zero initial condition is defined as $\sup_{\|w\|_2 \neq 0}(\|z\|_2/\|w\|_2)$, where $\|\cdot\|_2 = \sqrt{\int_0^\infty \|\cdot\|^2 dt}$ and $\|\cdot\|$ represent the L_2 norm and the Euclidian norm, respectively, for the vectors w and z.

2. System description

Consider a nonlinear descriptor system, having dynamics

$$E\frac{dx}{dt} = Ax + \varphi_1(x, u) + Bd, \tag{1}$$

$$y = Cx + H\varphi_2(x, u) + Dd, \tag{2}$$

where $u \in R^m$, $x \in R^n$, $y \in R^p$, and $d \in R^q$ represent the input, the state, the output, and the disturbance vectors for the above system. The nonlinear functions $\varphi_1(x, u) \in R^n$ and $\varphi_2(x, u) \in R^n$ denote the nonlinearities in the state and the output equations, respectively. The matrices $A \in R^{n\times n}$, $B \in R^{n\times q}$, $C \in R^{p\times n}$, $D \in R^{p\times q}$, $E \in R^{n\times n}$ and $H \in R^{p\times n}$ have constant entries. The matrix E, called the mass matrix, can be singular to represent a descriptor system. Further, the matrix E can be non-symmetric to denote a more general form of singular systems.

Remark 1. Usually, number of outputs is less than or equal to number of states for a system, that is $p \leq n$. If $p \leq n$ is verified, any arbitrary nonlinear function $f(x, u) \in R^p$ can be represented by $H\varphi_2(x, u)$, without loss of generality, through appropriate selection of H and $\varphi_2(x, u)$. Even if $p > n$, the nonlinearity $H\varphi_2(x, u)$ can represent $f(x, u)$, for instance, if some of the entries of $f(x, u)$ are zero.

Assumption 1.

(a) The nonlinear functions $\varphi_1(x, u)$ and $\varphi_2(x, u)$ satisfy the one-sided Lipschitz condition, given by:

$$\langle \varphi_i(x_1, u) - \varphi_i(x_2, u), x_1 - x_2\rangle \leq \rho_i\|x_1 - x_2\|^2, \quad i = 1, 2, \tag{3}$$

for real scalars ρ_1 and ρ_2 and $x_1, x_2 \in R^n$, where ρ_1 and ρ_2 are the one-sided Lipschitz constants for the corresponding nonlinearities.