Applied Mathematical Modelling 000 (2015) 1–12

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Analytic solution for maximum temperature during cut in and cut out in surface dry grinding

J.L. González–Santander*

Universidad Católica de Valencia "san Vicente mártir", C/Guillem de Castro 96, Valencia 46001, Spain

ARTICLE INFO

Article history:
Received 16 February 2015
Revised 25 June 2015
Accepted 22 September 2015
Available online xxx

Keywords:
Thermal damage
Surface dry grinding
Transient regime

ABSTRACT

Considering the Samara–Valencia model for heat transfer in surface dry grinding, analytical expressions are calculated for the time-dependent temperature field of the workpiece during the transient regime in which the wheel is engaged and disengaged from the workpiece. For this calculation, a linear heat flux profile along the contact zone between the wheel and the workpiece has been considered. From these results, a very rapid method is described for the numerical evaluation of maximum temperature at the initial and final workpiece edges. This analytical result is intended to be a very useful tool for the monitoring of the online grinding process in order to avoid the thermal damage of the workpiece.

1. Introduction

Surface grinding is a machining process of metallic plates, used for polishing its surface by using an abrasive wheel. The grinding wheel rotates at a high speed and slides over the workpiece surface, so that the surface material of the metallic plate being ground is removed. Most of the energy produced in grinding is converted into heat due to friction between wheel and workpiece, and this heat is accumulated within the contact zone between both [1,2]. The high temperatures reached during this machining process may produce an unacceptable decrease in the quality of workpieces, such as burning or residual stresses [3]. Recently in [4], it has been performed an analytical approach to speed up the numerical evaluation of maximum temperature in surface dry grinding in the stationary regime for any heat flux profile entering into the workpiece. However, the risk of thermal damage is higher in the transient regime occurring when the grinding wheel disengages from the workpiece (cut out) [5]. A practical way to avoid the latter is to clamp the workpiece between plates in order to provide heat conduction at its ends. However, plates should be of the same width as the workpiece, and also they have to be adjusted at the same level as the workpiece in order to control the depth of cut a (see Fig. 1), producing the latter a significant delay in the online process. Therefore, an analytical expression would be very useful for the maximum temperature prediction in this transient regime with the workpiece alone.

According to Fig. 1, the heat transfer in surface grinding is usually modeled [6] by a strip heat source infinitely long along the z-axis and of δ width (m in SI units), where the Cartesian coordinate system XYZ is fixed to the wheel. Notice that according to this modeling, the radius of the wheel should be much greater than the depth of cut, $R \gg a$. The workpiece consists of a semi-infinite solid that is moving with respect to the wheel at a speed of $\vec{v}_d = -v_d\,\vec{i}$ (m s^{-1}). The field of temperature rise in the workpiece

* Tel.: +34 963637412.
E-mail address: martinez.gonzalez@ucv.es

http://dx.doi.org/10.1016/j.apm.2015.09.031
S0307-904X(15)00565-X/

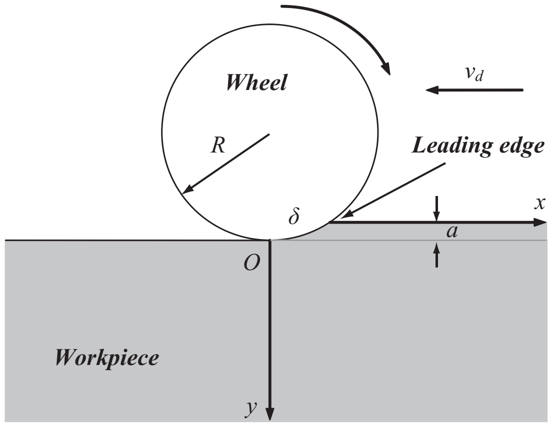

Fig. 1. Setup scheme in dry surface grinding.

$T(t, x, y)$ with respect to the room temperature T_0 (K) must satisfy the convective heat equation [7, §1.7(2)]

$$\frac{\partial T}{\partial t} = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) - v_d \frac{\partial T}{\partial x}, \tag{1}$$

where k is the thermal diffusivity ($m^2\ s^{-1}$). Since initially the workpiece is at room temperature T_0, (1) is subjected to an homogeneous initial condition

$$T(0, x, y) = 0. \tag{2}$$

The boundary condition is given by

$$k_0 \frac{\partial T}{\partial y}(t, x, 0) = b(t, x) T(x, 0, t) + d(t, x), \tag{3}$$

where k_0 ($W\ m^{-1} K^{-1}$) denotes the thermal conductivity, $b(t, x)$ is the heat transfer coefficient ($W\, m^{-2}\, K^{-1}$) between the workpiece and the environment, and $d(t, x)$ takes into account the heat flux ($W\, m^{-2}$) entering into the workpiece due to friction between wheel and workpiece. It is worth noting that both $b(t, x)$ and $d(t, x)$ are input functions in the model and they have to be determined by other means such as experimental measurements and a partition model for the energy generated by friction in the contact zone [8].

According to [10], the solution of the boundary-value problem stated in (1)-(3) can be split in two terms

$$T(t, x, y) = T^{(0)}(t, x, y) + T^{(1)}(t, x, y), \tag{4}$$

where

$$T^{(0)}(t, x, y) = -\frac{1}{4\pi k_0} \int_0^t \frac{1}{s} \exp\left(\frac{-y^2}{4ks}\right) \times \left\{ \int_{-\infty}^{\infty} d(t-s, \xi) \exp\left(-\frac{(\xi - x - v_d s)^2}{4ks}\right) d\xi \right\} ds, \tag{5}$$

and

$$T^{(1)}(t, x, y) = \frac{1}{4\pi} \int_0^t \frac{1}{s} \exp\left(\frac{-y^2}{4ks}\right) \left\{ \int_{-\infty}^{\infty} \left(\frac{y}{2ks} - \frac{b(t-s, \xi)}{k_0}\right) T(t-s, \xi, 0) \exp\left(-\frac{(\xi - x - v_d s)^2}{4ks}\right) d\xi \right\} ds. \tag{6}$$

Notice that $T^{(0)}(t, x, y)$ involves the part of the boundary condition (3) due to friction, and $T^{(1)}(t, x, y)$ the one due to convection. Notice as well that the time-dependent field of the temperature rise $T(t, x, y)$ given in (4) is an integral equation, since $T^{(1)}(t, x, y)$ involves the surface temperature rise $T(t, x, 0)$.

In the case of dry grinding (adiabatic case), the convection on the surface can be neglected, i.e. $b(t, x) = 0$, and the above integral equation can be solved as [11]

$$T(t, x, y) = 2T^{(0)}(t, x, y) \tag{7}$$

$$= -\frac{1}{2\pi k_0} \int_0^t \frac{1}{s} \exp\left(\frac{-y^2}{4ks}\right) \times \left\{ \int_{-\infty}^{\infty} d(t-s, \xi) \exp\left(-\frac{(\xi - x - v_d s)^2}{4ks}\right) d\xi \right\} ds. \tag{8}$$

The above result is termed as $T^{(0)}$ *theorem*.

Fig. 2 shows three sequential regimes during grinding for the analysis of the temperature field: cut in, stationary regime and cut out. During the transient regime of the cut in, the friction width increases approximately linearly from zero to its stationary