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Solutions of a crack interacting with a three-phase composite in plane elasticity

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ABSTRACT

The interaction between a crack and a circularly cylindrical layered media under a remote uniform load for plane elasticity is investigated. Based on the method of analytical continuation associated with the alternation technique, the solutions to the crack problem for a three-phase composite are derived. A rapidly convergent series solution for the stress field, expressed in terms of an explicit general term of the corresponding homogeneous potential, is obtained in an elegant form. The solution procedures for solving this problem consist of two parts. In the first part, the complex potential functions of dislocation interacting with a three-phase composite are obtained. In the second part, the derivation of logarithmic singular integral equations by introducing the complex potential functions of dislocation along the crack border is made. The stress intensity factors (SIFs) are then obtained numerically in terms of the dislocation density functions of the logarithmic singular integral equations. The stress intensity factors (SIFs) as a function of the dimensionless crack length for various material properties and geometric parameters are shown in graphic form. The obtained results may provide some guidance for material and geometry selections by minimizing the SIF.

1. Introduction

The fast advancement of multi-layered composites is now being widely used in engineering applications. Even though composites offer excellent performance, a small quantity of crack-like defects can cause a marked effect on the composite strength. In order to analyze the cracked composites, the evaluation of stress intensity factors (SIFs) at crack tips is usually used in the linear theory of fracture mechanics. SIF, which characterizes the singular stress field in the vicinity of the crack tips, is a key parameter in brittle fracture and substantial to predict fatigue life.

Singular Integral Equation (SIE) is one of the most extensively used methods in the evaluation of SIF for plane crack problems. In this approach, the fundamental solution of a dislocation would be required as a Green's function. By placing continuous distribution of dislocation along the prospective site of crack and together with superposition technique, the system of SIE is formulated. The basic idea of superposition technique is the sum of two solutions: the first obtained for the given external loads and the given medium without the crack, and the second obtained for the medium with a crack, so that the crack faces become traction free. The system of SIE is then solved numerically to calculate the SIF. There are two major approaches to obtain SIE, namely integral transform [1] and Muskhelishvili's complex potential [2]. Depending on the crack geometry and the coordinate

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system, Fourier and Mellin integral transforms [3] are the most frequently used to reduce the mixed boundary-value problem to SIE. However their method requires some inverse transform procedure, which is somewhat cumbersome. The complex potential in conjunction with analytical continuation technique [2] is relatively simplified compared to the method using integral transforms. The method of complex potentials provides perhaps the simplest method to analyze the plane crack problem, particularly in problems involving multi-layered materials. Comparatively speaking, the Muskhelishvili complex potential function provides a flexible way to solve the crack problem [4,5]. With this in mind, the singular integral equation based on the Muskhelishvili complex potentials is more effective.

A new integral equation with logarithmic singular kernels in the crack problem was first proposed by Cheung and Chen [6]. The formulation differs from the others in that the resultant force function is used in the integral equation representation rather than the traction. This leads to a logarithmic singular kernel. Different from Cauchy singular kernels which preserve a strong singularity, the development for weakly singular (logarithmic singular) allows easy calculations in the singular integral equation. A number of studies for solving crack problems has been developed using logarithmic singular in conjunction with complex potential. They include plane elasticity problems with branch crack [7], curve crack [8], crack in elastic half-plane [9], circular crack in bonded dissimilar materials [10], curve crack interacted with elastic inclusion [11], thermal crack in bonded dissimilar media [12], and a line crack interacted with circular inclusion [13]. All the above mentioned studies are restricted to the problem containing cracks in two bonded half-plane media or a single inclusion embedded in an infinite matrix which can be expressed in closed form solution in terms of point dislocation. However, there are many multiphase systems and composite materials that are commonly encountered in engineering application. Physically, composite materials can be defined as a heterogeneous mixture of two or more homogeneous phases that have been bonded together. Because of that, when a crack is present in multi-layered composites, the solution must satisfy the continuity conditions along different interfaces. Many investigators have obtained the solution by using integral transforms and Laurent series [14–16]. However, the application is very difficult for multiply connected region involved instead of a simply connected domain. In order to overcome the difficulty, application of complex potential in conjunction with the analytical continuation technique [17,18], that is alternately applied across two different interfaces in order to derive the dislocation solution in a series form, has been introduced [19,20]. The merit of this method is that the analytical continuation technique allows us to easily deal with the interface continuity conditions of multi-layered composites. Besides, the derived series solution is in terms of the material parameters instead of the distance between singularities and the interfaces, making the series solution rapidly convergent even a dislocation is located closer to the interface. This method has been successfully applied into several problems of singularities interacting with multi-layered media, such as singularity in isotropic tri-material [19], isotropic tri-material interacted with a point heat source [21], an arbitrary point force in circularly cylindrical layered media [22], a point heat source in circularly cylindrical layered media [23], an edge dislocation interacted with coated elliptic inclusion [24], and a coated circular inclusion under uniaxial tension [25].

In the present study, the interaction between a crack and multi-layered composites for plane elasticity is considered. The study can be achieved by the determination of the stress intensity factors (SIFs) that allow the characterization of interaction from the point of view of linear elastic fracture mechanics. The proposed method for solving this problem consists of two parts. In the first part, based on the method of analytical continuation in conjunction with the alternating technique, the complex potential functions of dislocation interacting with multi-layered composites are obtained. The second part consists of the derivation of logarithmic singular integral equations by introducing the dislocation functions along the crack border together with superposition technique. The singular integral equation is then solved numerically by modeling a crack in place of several segments. Linear interpolation formulae with undetermined coefficients are applied to approximate the dislocation distribution along the elements, except at vicinity of crack tip where the dislocation distribution preserves a square-root singularity. Once the undetermined dislocation coefficients are solved, the stress intensity factors (SIFs) can be obtained.

The layout of the present work is as follows. Following this brief introduction in Section 1, the basic principle concerning complex potential functions is briefly given in Section 2. The derivation of Green's function for edge dislocation located either in an infinite matrix or in core region is provided in Section 3. The establishment of singular integral equations is provided in Section 4. Several numerical examples are discussed in Section 5. Finally Section 6 concludes the article.

2. Problem formulation

Schematically illustrated in Fig. 1 is a circularly cylindrical layered media interacted with a crack located either in an infinite matrix or in core inclusion. Let S_1 denote the infinite matrix, S_2 denote the coating layer, and S_3 denote the core inclusion, respectively. The boundaries of coating layer are two circles Γ_1 and Γ_2 with radius R and r respectively, which are assumed to be perfect, i.e. both tractions and displacements are continuous across the two interfaces. Let the circularly cylindrical layered media contain a line crack with length $2a$ located in an infinite matrix S_1 or in core inclusion S_3 with distance h from the circle interface and subjected to a remote uniform tension T.

Based on the complex variable theory for a two-dimensional plane elasticity, the component of displacement $u + iv$ and resultant forces $-Y + iX$ can be described by two complex functions $\phi(z)$ and $\omega(z)$, each of which is analytic in its argument $z = x + iy$ [26], as

$$2\mu(u+iv) = \kappa\phi(z) - \overline{\omega(z)} + (\bar{z}-z)\overline{\phi'(z)}$$
$$-Y+iX = \phi(z) + \overline{\omega(z)} + (z-\bar{z})\overline{\phi'(z)}$$