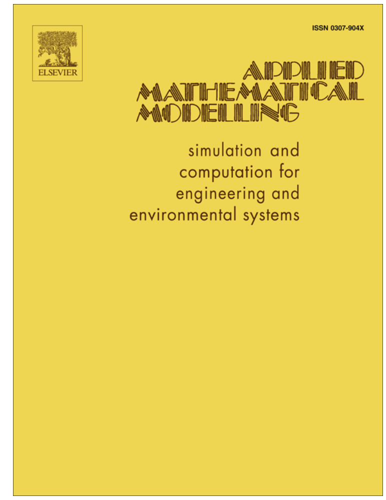

Limited Memory Technique using Trust Regions for Nonlinear Equations

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Limited Memory Technique using Trust Regions for Nonlinear Equations

Linghua Huang *

Abstract A new trust region subproblem that accelerates convergence for solving symmetric nonlinear equations is defined. To avoid repeatedly computing the trust region subproblem, a line search technique without derivative information is used in the trust region algorithm. Moreover, a limited memory BFGS update is employed to generate an approximated matrix rather than a normal Jacobian matrix or quasi-Newton matrix. Under mild conditions, the global convergence and the superlinear convergence of the given algorithm are established. The numerical results indicate that this method can be beneficial for solving the presented problems.

Keywords: Limited Memory BFGS method; trust region method; nonlinear equations; line search; global convergence.

AMS 2010 subject classifications. 90C26.

1. Introduction

Consider

$$N(x) = 0, \; x \in \Re^n \tag{1.1}$$

where $N : \Re^n \to \Re^n$ is continuously differentiable and the Jacobian $\nabla N(x)$ of N is symmetric for all $x \in \Re^n$. Let ϕ be the norm function defined by $\phi(x) = \frac{1}{2}\|N(x)\|^2$, where $\|\cdot\|$ is the Euclidean norm. Then, the nonlinear system of equations (1.1) is equivalent to the following global optimisation problem:

$$\min \phi(x), \; x \in \Re^n. \tag{1.2}$$

Throughout this paper, $N(x_k)$ is replaced by N_k, x_k is the k-th iterative point, and the Jacobian matrix $\nabla N(x_k)$ of $N(x)$ at x_k is replaced by ∇N_k. The trust region (TR) method is one of the most effective methods for solving (1.1) because it always exhibits good convergence. Traditional trust region methods, when applied at each iterative point x_k, yield the trial step d_k through the following equation, which is called a TR subproblem:

$$\begin{aligned} \min_{d \in \Re^n} q_k(d) \quad &= \quad \frac{1}{2}\|N_k + \nabla N_k d\|^2 \\ s.t. \quad & \|d\| \leq \triangle_k, \end{aligned} \tag{1.3}$$

where $\triangle_k > 0$ is a scalar called the trust region radius. If computing the Jacobian matrix $\nabla N(x_k)$ is computationally expensive, a commonly used method consists of using a quasi-Newton matrix (such as BFGS or DFP) rather than ∇N_k. Yuan et al. [23] presented a BFGS method defined by

$$\begin{aligned} \min_{d \in \Re^n} q_k(d) \quad &= \quad \frac{1}{2}\|N_k + A_k d\|^2 \\ s.t. \quad & \|d\| \leq \triangle_k, \end{aligned} \tag{1.4}$$

where $\triangle_k = c^p\|N_k\|$, $c \in (0,1)$, $p > 0$ is an integer and A_k is generated by the BFGS formula

$$A_{k+1} = A_k - \frac{A_k s_k s_k^T A_k}{s_k^T A_k s_k} + \frac{y_k {y_k}^T}{{y_k}^T s_k}, \tag{1.5}$$

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