ARTICLE IN PRESS

Applied Mathematical Modelling xxx (2015) xxx–xxx

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Approximate method for stress intensity factors determination in case of multiple site damage

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ARTICLE INFO

Article history:
Received 20 September 2013
Received in revised form 24 November 2014
Accepted 12 January 2015
Available online xxxx

Keywords:
Stress intensity factor
Multiple site damage
Finite element method
Approximate procedure
Interaction between multiple crack tips

ABSTRACT

A simple and easy to use approximate procedure, for calculating stress intensity factors, was proposed. The procedure was developed based on existing solution for stress intensity factor in the case of two unequal cracks in an infinite plate subjected to remote uniform stress. The solution for this configuration was used for obtaining interaction effect coefficients which take into consideration the increase of stress intensity factor of analyzed crack tip due to interaction with existing adjacent crack. Accuracy and application of suggested procedure were verified through two different computer programs which are based on two different computational methods: finite element method (FEM) with singularity elements and extended finite element method (X-FEM). The analysis of the results has shown that a very good agreement between solutions was achieved, and that this method can provide stress intensity factors with acceptable accuracy.

1. Introduction

The continuous integrity of supporting structures of aging aircrafts is of great concern to the aviation community. The long service life of aging aircrafts increases the possibility of diminishing, or even total loss of structural integrity due to multiple fatigue cracks. Multiple site damage (MSD) represents the simultaneous development of fatigue cracks at multiple sites in the same structural element. Those cracks are close enough to influence each other and to affect the overall structural integrity. MSD often occurs in longitudinal and circumferential riveted lap joints in wings and fuselages. It can be very serious, because of possible link up of adjacent cracks creating one large crack that can cause catastrophic failure, due to reduction of residual strength of structural element.

The prediction of crack growth rate and residual strength of cracked structure demands accurate calculation of stress intensity factors (SIFs). In order to predict those factors, several analytical and theoretical studies, as well as numerical models and have been presented over the years.

Analytical procedure had been presented in which the stress function was assumed to be the sum of stress functions with singularities on the crack faces and at the infinity and the stress intensity factors for various configurations of two-dimensional interacting cracks in an infinite body subjected to a remote tension were given as a power series formula [1]. Theoretical analysis followed which enabled formation of an approximate expansion polynomial expression for the stress intensity factor [2].

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http://dx.doi.org/10.1016/j.apm.2015.01.050
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The analytical method introduced for solving stress intensity factor problems on multiple holes by Zhao et al. [3]. This modified analytical method is easier to apply than some traditional analytical methods.

The Schwartz–Neumann alternating method, together with the boundary element method was used to determine the mixed mode stress intensity factors and weight functions for cracks in finite bodies [4], while an alternating indirect boundary element (AIBE) technique was used to calculate stress intensity factors for multiple interacting cracks in two-dimensional cracked structure by Dawicke and Newman [5].

A two-dimensional plane stress elastic fracture mechanics analysis of a clap joint fastened by rigid pins was performed, where two types of MSD are considered: MSD with equal length cracks and MSD with variable crack length and the plate treated as being of infinite thickness. The mode I stress intensity factors and changes in the compliance due to the existence of MSD were determined in paper presented by Beuth and Hutchinson [6].

Hybrid finite element method was used, together with complex variable theory of elasticity to calculate the stress intensity factor at the crack tips, stress concentration factors in the stiffeners, and rivet loads for a stiffened structure with multiple cracks [7].

Regardless of the mentioned research, there is still a lack of available solutions in case of more complex configurations with more than few cracks. The solutions for these configurations, now more than ever, imply the usage of numerical methods, as technology and computer sciences became more available. Nevertheless, this kind of analysis can be very complicated especially because of the mutual influence of the adjacent cracks. This is the main reason for introducing approximation methods and procedures which will enable faster and simpler determination of stress intensity factors of supporting aircraft structures with multiple cracks, but they were very occasional the topic of the researcher's studies. One of the rare methods of this kind was a compounding method for determining approximate stress intensity factors which is performed by adding individual boundary effects, presented by Cartwright and Rooke [8]. But, the evaluation of the interaction between boundaries effect was very difficult, since it increases with the increase of boundary number and with the crack tip approaching to the boundary. This significantly influenced the accuracy of the method. This method was used for the assessment of influence of the adjacent cracks for calculating the stress intensity factor in the case of multiple elliptical and through cracks that develop from adjacent rivet holes of a thin plate by Pastrama and De Castro [9]. A simple method of stress analysis in elastic solids with many cracks was proposed by Kachanov [10]. It was based on the superposition technique and the ideas of self-consistency applied to the average tractions on individual cracks. This method was specialized to arbitrary length collinear cracks with arbitrary spacing under far field tension by Millwater [11].

However, the assessment of mutual influence of the adjacent cracks remains one of the major problems for approximate methods.

In this paper a versatile and easy to use approximate procedure for stress intensity factor determination in case of multiple cracks is presented. This procedure takes into consideration the effect of the interaction between multiple crack tips as corresponding coefficients, which represent the influences of adjacent cracks on stress intensity factor of analyzed crack. The accuracy of the procedure was verified by comparison with the solutions obtained with finite element method and extended finite element method.

2. Approximate method for stress intensity factor determination in case of MSD

The procedure was developed based on existing solutions for stress intensity factors in the case of two unequal cracks in an infinite plate subjected to remote uniform stress [12].

For this model, normalized SIFs for crack tips A, B, C i D are calculated with following expressions:

$$\beta_A = \frac{K_{IA}}{K_{01}} = \sqrt{\frac{2b}{a_1}} \cdot \frac{x_A^2 - C_1 x_A + C_2}{\sqrt{x_A(x_A + x_C)(x_A + x_D)}}, \tag{1}$$

$$\beta_B = \frac{K_{IB}}{K_{01}} = \sqrt{\frac{2b}{a_1}} \cdot \frac{C_2}{\sqrt{x_A \cdot x_C \cdot x_D}}, \tag{2}$$

$$\beta_C = \frac{K_{IC}}{K_{02}} = \sqrt{\frac{2b}{a_2}} \cdot \frac{x_C^2 + C_1 x_C + C_2}{\sqrt{x_C(x_A + x_C)(x_D - x_C)}}, \tag{3}$$

$$\beta_D = \frac{K_{ID}}{K_{02}} = \sqrt{\frac{2b}{a_2}} \cdot \frac{x_D^2 + C_1 x_D + C_2}{\sqrt{x_D(x_A + x_D)(x_D - x_C)}}, \tag{4}$$

where:

$K_{01} = \sigma\sqrt{\pi a_1}$, for crack tips A i B, and $K_{02} = \sigma\sqrt{\pi a_2}$ for crack tips C i D.

Those normalized SIFs for opening mode are given as a function of dimensionless parameters x_A, x_C and x_D, which can be expressed as: