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Coherence-enhancing diffusion with the source term

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ABSTRACT

Many texture images require the enhancement of coherent structures in various applications. Traditional coherence-enhancing diffusion filtering (CED) completes the interrupted lines and gaps but at the cost of reducing the contrast between coherent structures and the background. In this study, we introduce a source term into CED filtering to restore the initial image and the contrast lost by pure diffusion filters. Moreover, this new model combines contrast enhancement and diffusion processes, so it may be more suitable for dealing with white noise than the original CED. We assessed our method in terms of the theoretical and numerical properties changed by the source term. In our numerical assessment, we implemented our approach using an explicit scheme, which was accelerated by fast explicit diffusion. We compared the performance of our proposed approach with CED filtering based on fingerprint images.

1. Introduction

Various texture images surround us in our daily lives, which range from fingerprint images to cone beam CT data in medical applications [1]. Handling texture image is a challenging task for both computer vision and human vision. For example, enhancing poor quality fingerprint images with noise degradation is important in forensic applications as well as in commercial security application, where this requirement is growing rapidly.

1.1. Nonlinear diffusion

Recently, numerous approaches have been suggested for dealing with texture images (e.g., see [2] and the references therein), including methods based on partial differential equations (PDEs), where filters formulated using nonlinear PDEs have obtained impressive results [3]. In 1990, Perona and Malik [4] initiated the construction of nonlinear diffusion methods (for review articles, see [3,5]), most of which use edge detectors to abstract local information in the diffusion process to preserve edges and remove noise. However, because diffusivity is scalar, rather than a diffusion tensor, traditional nonlinear diffusion methods produce less satisfactory results with texture images.

The idea of anisotropic diffusion filtering was pioneered by Nitzberg et al. [6] and Cottet et al. [7]. Later, Weickert proposed coherence-enhancing diffusion (CED) filtering in 1999, the basic idea of which is to direct nonlinear diffusion using a structure tensor [8–12], instead of employing scalar-valued diffusivity to smooth and enhance the texture structure in the coherent orientation. CED obtains impressive results when dealing with texture images compared with other nonlinear diffusion filtering methods [13,14]. The CED approach was improved by introducing an optimized rotation invariant scheme

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[15], an iterative scheme [16], and fast explicit diffusion (FED) [17]. By combining the sharpening qualities of shock filters with the structure tensor, Weickert proposed a novel method called coherence-enhancing shock filters for enhancing the coherent flow-like structures in texture images [18]. In [19], a method called CED-OS was introduced by Franken and Duits for the enhancement of elongated structures. Moreover, Obara et al. replaced the structure tensor with a phase congruency tensor, which facilitated the successful enhancement of noisy texture images [20].

1.2. CED filtering

Next, we provide an outline of the rationale of CED filtering. CED combines nonlinear diffusion filtering with orientation analysis using the so-called structure tensor. The principle of this method is as follows:

$$\begin{aligned}&\partial_t u = \operatorname{div}(D\nabla u), \quad \text{on} \quad \Omega \times (0,T),\\ &(D\nabla u, n) = 0, \quad \text{on} \quad \partial\Omega \times (0,T),\\ &u(x,0) = f(x), \quad \text{on} \quad \Omega,\end{aligned}$$

where f is the initial image, $\Omega \in R^2$ is a two-dimensional domain (2-D), n denotes the outer normal, and $(.,.)$ is the usual inner product.

As a function of local image structures, the diffusivity $D(J_\rho(\nabla u_\sigma))$ is constructed as follows [21]. First, u is convolved with a Gaussian kernel $k_\sigma : u_\sigma = k_\sigma * u$ to ignore any false information due to noise. ∇u_σ is unsuitable for finding parallel structures in texture images [14], so the structure descriptor is replaced by the tensor product of $\nabla u_\sigma : \nabla u_\sigma \otimes \nabla u_\sigma = \nabla u_\sigma \nabla u_\sigma^T$. The local information is then averaged by convolving component-wise over $\nabla u_\sigma \nabla u_\sigma^T$ with a Gaussian kernel k_ρ, thereby yielding a symmetric, positive semidefinite matrix $J_\rho(\nabla u_\sigma) = k_\rho * \nabla u_\sigma \nabla u_\sigma^T$. The orthonormal eigenvectors of the matrix indicate the orientation that maximizes the gray-value fluctuations to give the preferred local direction of smoothing.

For texture images, where we need to complete the interrupted lines and preserve the edges, we should smooth mainly in the coherent direction v_2 and the diffusivity should be adapted to the coherence strength of its orientation. This can be achieved using the following construction of D, which has the same eigenvector as J_ρ and its eigenvalues are given by

$$\begin{aligned}&\lambda_1 = \alpha\\ &\lambda_2 = \begin{cases}\alpha & \text{if} \quad \kappa = 0\\ \alpha + (1-\alpha)e^{-\frac{C}{\kappa}} & \text{else}\end{cases},\end{aligned}$$

where $C > 0$ is a threshold, κ is a measure of coherence, and $\alpha \in (0,1)$ keeps the diffusion tensor as uniformly positive definite [22].

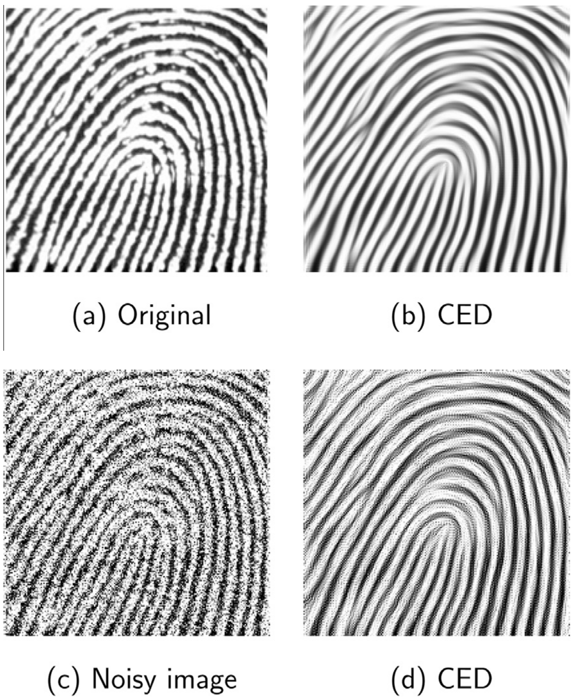

Fig. 1. Results obtained after applying CED. (a) Original image; (b) CED filtering with $dt = 0.1, \sigma = 1, \rho = 7, N = 300$; (c) original image with Gaussian noise where the deviation is 50; (d) CED filtering with $dt = 0.1, \sigma = 1, \rho = 7, N = 300$.

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