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Modeling the arterial wall mechanics using a novel high-order viscoelastic fractional element

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ABSTRACT

The fractional viscoelastic models (FVMs) have provided promising results for modeling the behavior of complex materials such as polymers and living tissues. These viscoelastic models are composed by springs, dashpots and the fractional element called spring-pot. In this paper we prove that the accuracy of these models can be improved through the use of a modified version of the spring-pot element, called high-order spring-pot (HOSP).

We describe and implement a numerical method for characterization of mechanical properties of FVMs. The method consists of minimizing the misfit among experimental measures of strains or stresses and the respective values predicted by the model. The method is validated by solving four numerical examples. In the first three examples the data is artificially generated using different models such as the Double Maxwell-arm Wiechert one. The characterization is performed using FVMs models including the traditional spring-pot element and the new HOSP element proposed in this article. In these examples we assume small strains and homogeneous material properties. In a final example the method is applied to the characterization of the mechanical properties of FVMs using stress–strain data obtained from *in vitro* ovine arterial wall measurements reported in the literature.

The results obtained show that the proposed method properly determines the mechanical parameters even in presence of noise in the data. In addition, it is evident from the results that the proposed modification of the spring-pot element increases the accuracy of the FVMs models. The results obtained allow us to conclude that the FVMs can model better the behavior of complex materials when a HOSP element is included. In particular, it was shown that these models are appropriate for modeling the arterial wall mechanics with higher accuracy, as well as other materials with complex behavior.

1. Introduction

The behavior of viscoelastic materials is usually represented by simple models composed by elastic and viscous elements, which may be linear springs and dashpots, respectively. These rheological models have been widely used for modeling the behavior of several materials, such as concrete [1]. In [2,3] it is pointed out that living tissues behavior should be generally considered viscoelastic, and we can find in the literature applications of these models to the modeling of a broad set of living tissues [4–6]. There exists a particular interest in the modeling of the arterial wall behavior, motivated by the relevance of

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the Atherosclerotic Cardiovascular Disease (ACD) in the human death toll [7]. Viscoelastic models have been widely applied to the arterial wall mechanics modeling [8,9]. However, Lakes [10] pointed out that simple models composed by a small number of springs and dashpots must be used only with a pedagogic purpose, since real materials are not usually describable by these models. This statement have been confirmed by Wang et al. [6] who have shown that by increasing the number of elements ones improves the accuracy of the rheological models applied to the modeling of living tissues.

We note also that there are many papers in the literature dealing with the development of complex nonlinear elastic models for arterial wall modeling. In particular, the arterial wall tissue is anisotropic, and has been modeled considering histological information such as the fiber orientation in each layer [11]. Recently, nonlinear effects have also been considered in viscoelastic models [12]. In this article we do not consider neither geometric nor material nonlinearities.

Fractional viscoelastic models (FVMs) are those that include at least one fractional element, also known as spring-pot. The spring-pot element represents an intermediate behavior between a linear spring and a linear dashpot. This intermediate behavior is mathematically represented by defining the stress proportional to the fractional derivative of the strain, i.e. $\sigma \propto D^{\alpha}\varepsilon$, where the derivative order α is considered another mechanical parameter, called fractional parameter. To the authors knowledge, in all applications of FVMs considered in the literature, the fractional parameter is a real number within the interval $[0, 1]$.

The first applications of fractional models were reported in the early twentieth century [13]. Throughout the years the interest in the matter have been increasing and important aspects were improved such as new mathematical definitions of the fractional derivative with appropriate physical interpretations [14,15]. In the 80's, Bagley and Torvik [16] presented links between molecular theories and macroscopic fractional viscoelastic models. Since then, they and other authors [17,18] have contributed to the determination of algebraic constraints for the material parameters of the models taking into account thermodynamics constraints.

FVMs are recognized to be well suited for modeling the behavior of real complex materials such as anisotropic structural elements [19], polymers [20,21] and chemical compounds [14]. For living tissues, the FVMs have been successfully applied to the characterization of cells mechanical properties [22], as well as the modeling of the arterial wall mechanics [23–25].

The use of several springs and dashpots in certain arrangements provides constitutive equations with integer derivative orders higher than one. Recently, it was shown that the use of these models produces accurate results [26]. In this paper we use a fractional element that we call high-order spring-pot (HOSP), which is defined by setting the upper bound of the fractional parameter equal to 2. We use FVMs with one HOSP and prove its ability to reproduce the behavior of complex models with higher accuracy. In Appendix B, it is presented a thermodynamic justification of this new upper bound.

In Section 3 we describe a method for characterization of mechanical parameters of FVMs, including the fractional parameter. The numerical method obtains the parameters by solving a nonlinear optimization problem, using a fitness function that depends on the coefficients of the discrete Fourier transform of the given data.

In Section 4 a validation of the proposed method that is performed by studying some examples with numerically generated and experimental data is presented. The experimental data used in this paper is part of the data presented in [9], and consists of measurements of pressure and diameter of arterial wall segments of healthy sheeps. The results obtained show that the HOSP provides higher accuracy in the modeling of the behavior of complex materials when compared against the traditional spring-pot.

Preliminary results of this work were presented in [27]. A more detailed description of the characterization method is presented here, and the theoretical justification about the use of the HOSP element is published in this paper for the first time. Some new insightful examples are presented, as well as a deeper discussion of the results and the advantages of the HOSP element over the traditional spring-pot for the modeling of living tissues.

2. Preliminaries

In this section we present the basic concepts of fractional calculus, some important properties of the Fourier transform and the three viscoelastic models used through the article.

2.1. Fractional calculus

The fractional calculus theory introduces and studies several possible generalizations of the concept of derivative of a function of real number or even complex number order. During the twentieth century it has been used as an appropriate approach for modeling the behavior of viscoelastic materials [16]. In this field, the derivatives of a real number order, also called fractional derivatives, are used to define structural elements for which the stress response is intermediate between that of a spring and that of a dashpot, i.e. the responses corresponding to a zero or a first order derivative of the strain, respectively.

There are different possible definitions for the fractional derivatives of a function, in this paper we will consider the Riemann–Liouville definition with lower terminal as $-\infty$, given by the following expression:

$$D^{\alpha}f(t) = \frac{1}{\Gamma(n-\alpha)} \int_{-\infty}^{t} \frac{f^{(n)}(\tau)}{(t-\tau)^{\alpha+1-n}}\, d\tau \quad n-1 < \alpha < n. \tag{1}$$

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