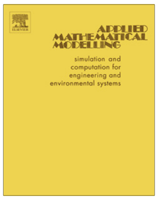
ARTICLE IN PRESS

Applied Mathematical Modelling xxx (2015) xxx–xxx

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm



Free vibration analysis of composite, circular annular membranes using wave propagation approach

Arian Bahrami *, Amir Teimourian

Department of Mechanical Engineering, Eastern Mediterranean University, G. Magosa, TRNC, Mersin 10, Turkey

A R T I C L E I N F O

Article history:
Received 21 December 2013
Received in revised form 12 March 2015
Accepted 20 March 2015
Available online xxxx

Keywords:
Composite membranes
Natural frequency
Propagation matrix
Reflection matrix
Transmission matrix
Vibration analysis

A B S T R A C T

This paper presents the wave propagation approach for free vibration analysis of non-uniform annular and circular membranes. Literature reviews reveal that most bodies analyzed by this approach are one dimensional waveguide structures. From wave standpoint, vibration propagates, reflects and transmits in a structure. Firstly, the propagation, reflection and transmission matrices for non-uniform annular and circular membranes are derived. Then, these matrices are combined to provide a concise and systematic approach for obtaining the natural frequencies of non-uniform annular and circular membranes. The solution obtained by this approach is exactly the same as those derived by the classical method. Moreover, a set of benchmark results is presented for various geometric parameters. Finally, the behavior of propagation, reflection and transmission matrices is studied by defining their important parameters. The obtained hints are useful for the analysis of energy transmission in micro/nano devices.

1. Introduction

The vibration of membrane is a subject of scientific interest. In a non-homogeneous membrane, the membrane has variable thickness or material density. The vibration analysis of circular and annular non-homogeneous membranes were investigated using approximate and exact methods by several authors. Gutierrez et al. [1] gave solutions for the vibration frequencies of linear, parabolic and cubic variations of density using four approximate methods: optimized Rayleigh–Ritz, differential quadrature, finite elements, and lower-bound solution based on the Stodola–Vianello method. Laura et al. [2] investigated the vibration of a composite annular membrane with linear density variation in the radial direction using two approximate approaches such as Rayleigh–Ritz and differential quadrature methods. Moreover, Laura et al. [3] presented an exact solution for the axisymmetric vibration modes of composite, circular annular membranes. Later, Rossit et al. [4] added the exact vibration frequencies for the antisymmetric modes for composite doubly-connected circular membranes. Jabareen and Eisenberger [5] presented exact solutions using a power series solution for both the axisymmetric and antisymmetric modes of circular and annular membranes with any piecewise polynomial variation of the density. Wang [6] found that the non-homogeneous rectangular membrane with linear density variation has a closed form exact solution. Also, he presented exact solution for the vibration of a continuously non-homogeneous annular membrane. Ersoy et al. [7] applied differential quadrature method for frequency analysis of membranes having irregular domains using an eight-node curvilinear element [7]. Moreover, he used the method of discrete singular convolution to study the free vibration of circular and

---

* Corresponding author. Tel.: +90 392630 2598; fax: +90 392365 3715.
E-mail address: arian.bahrami@emu.edu.tr (A. Bahrami).

http://dx.doi.org/10.1016/j.apm.2015.03.057

Please cite this article in press as: A. Bahrami, A. Teimourian, Free vibration analysis of composite, circular annular membranes using wave propagation approach, Appl. Math. Modell. (2015), http://dx.doi.org/10.1016/j.apm.2015.03.057

annular membranes with varying density [8]. Besides the mentioned methods, there is an alternative method known as wave propagation method. It is a simple, non-iterative and efficient method for calculating the natural frequencies of the structures. Unlike other methods, the wave propagation approach can also provide analytical expressions for transmitted, reflected power and energy flow of the waves in the waveguides using the derived reflection and transmission coefficients. As a result, the obtained hints can be used in transferring the desired energy for industrial applications if the environment behaves like that structure. Wave propagation, transmission and reflection in solids have been investigated by a number of researchers. The initial references for the wave propagation in solid bodies are Achenbach's [9] and Graff's [10] books which mostly focus on one dimensional bodies such as rods and beams. The first good research was done by Mace [11] who studied the free vibrations of Euler Bernoulli beams using the wave propagation approach. The propagation of elastic wave in spinning Timoshenko beam was then investigated by Argento and Scott [12]. In 1998, Tan and Kang [13] applied the wave propagation approach for an axially tensioned, rotating Timoshenko shaft. In 2001, Zhang et al. [14] studied the vibration of thin cylindrical shells using a wave approach based on Love's thin shell theory to calculate the natural frequencies. Also, the authors applied this approach for coupled vibration of fluid-filled shells [15], cross-ply laminated composite shells [16], and submerged shells [17]. In 2005, Mei and Mace [18] presented the wave propagation approach in a Timoshenko beam with discontinuity in the length of the beam. Also, the free and forced vibrations of axially loaded cracked Timoshenko beams were analyzed by Mei et al. [19] using the wave approach. In 2007, the wave method was proposed by Lee et al. [20] for thin, uniform, and curved beams with constant curvature to calculate the frequencies of the curved beams. They also considered the wave approach for analyzing the non-uniform waveguides such as non-uniform bars and non-uniform Euler Bernoulli beams whose properties vary rapidly but deterministically [21]. In 2008, Bahrami and Loghmani [22] presented the wave approach for the non-uniform rods with exponentially varying cross-sections. Moreover, Xuebin [23] analyzed the free vibration of a circular cylindrical shell using wave propagation approach based on Flugge's thin shell theory. In 2010, an exact wave-based analytical solution was presented by Mei [24] for obtaining the natural frequencies of classical planar frame structures, in which the coupling effect between bending and longitudinal vibrations was taken into account. In 2012, Mei [25] used a wave vibration approach to study the effects of lumped end mass on bending vibrations of a Timoshenko beam. Mei [26] applied the wave approach to obtain the natural frequencies and mode shapes of single-story multi-bay planar frame structures. Furthermore, Mei [27] analyzed the vibration of single-story multi-bay planar frame structures using the wave approach, in which the effects of rotary inertia, shear deformation, and joint model on vibration characteristics were taken into account. Most of the mentioned references used the wave propagation approach in one dimensional waveguides or planar frame structures, in which the waves propagate along one dimensional structural elements and are reflected and transmitted at joints and boundaries. To the best knowledge of the authors, there has been only one attempt to apply the wave approach for two dimensional uniform waveguide structures. Recently, Bahrami et al. [28] used the wave approach for uniform circular membranes and sectorial membranes to obtain the natural frequencies of these structures. There have been no attempts to apply the wave approach for analyzing two dimensional non-uniform waveguide structures such as non-uniform circular membranes and plates in which the use of Bessel functions is inevitable. So, a simple analytical wave approach for analysis of these structures seems essential due to the practical usage of these structures in the industry.

In this paper, the wave propagation approach is used to analyze the free vibration of non-uniform annular circular membranes. In Sections 2 and 3, the motion equation of non-uniform circular membranes and its solutions are presented, respectively. In Section 4, the propagation, reflection and transmission matrices are obtained for non-uniform annular circular membranes. In Section 5, the wave propagation approach is used to obtain the natural frequencies of the membranes. Finally, in Section 6, the obtained natural frequencies are compared with the results of the classical method and the behavior of propagation, reflection, transmission matrices is studied which is useful for the analysis of energy transmission in micro/-nano devices.

2. Formulation of the problem

Consider a composite membrane with two segments as shown in Fig. 1. Based on the small deflection theory, the motion equation of a non-uniform vibrating membrane with two segments as shown in Fig. 1 is defined as:

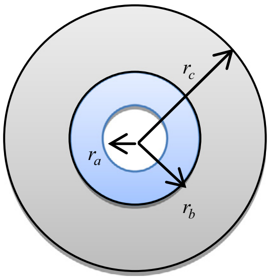


Fig. 1. A non-uniform annular membrane.

Please cite this article in press as: A. Bahrami, A. Teimourian, Free vibration analysis of composite, circular annular membranes using wave propagation approach, Appl. Math. Modell. (2015), http://dx.doi.org/10.1016/j.apm.2015.03.057