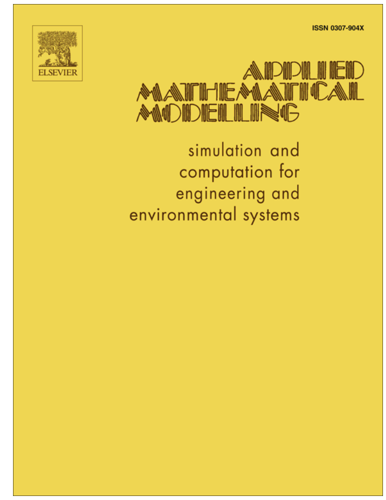
Accepted Manuscript



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PII: S0307-904X(15)00252-8
DOI: http://dx.doi.org/10.1016/j.apm.2015.04.016
Reference: APM 10545

To appear in: *Appl. Math. Modelling*

Received Date: 7 March 2014
Revised Date: 26 February 2015
Accepted Date: 13 April 2015

Please cite this article as: J. Lv, K. Wang, J. Jiao, Stability of stochastic Richards growth model, *Appl. Math. Modelling* (2015), doi: http://dx.doi.org/10.1016/j.apm.2015.04.016

Stability of stochastic Richards growth model

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Abstract

This paper is devoted to asymptotic analysis of equilibrium states of stochastic Richards equation. Both Itô and Stratonovič interpretations are investigated. Sufficient conditions for asymptotical stability of the zero solution and the positive equilibrium are established. Numerical simulations are introduced to illustrate the main results.

Keywords: Richards model, Brownian motion, Global asymptotical stability.

1. Introduction

Logistic model has been studied extensively owing to its theoretical and practical significance. The generalized logistic equation (Richards model[1, 2]) has the form

$$dx(t)/dt = rx(t)\left(1 - x^{\theta}(t)/K\right), \tag{1}$$

where $x(t)$ denotes the population size, r is the growth rate, θ is a positive constant and $K > 0$ is the carrying capacity. Obviously, if $\theta = 1$, model(1) is the classical logistic model. For the classical logistic model, a famous result is that if $r < 0$ and $0 \le x_0 < \sqrt[\theta]{K}$, then $\lim_{t\to+\infty} x(t) = 0$; if $r > 0$, then $\lim_{t\to+\infty} x(t) = K$ (see e.g. Murray [3]). As a matter of fact, the natural growth of many populations vary with t, for example, due to the seasonality. Therefore it is reasonable to consider the nonautonomous equation

$$dx(t)/dt = r(t)x(t)\left(1 - x^{\theta}(t)/K\right), \tag{2}$$

where $r(t)$ is bounded and continuous function on $[0, +\infty)$.

In the real world, population systems are inevitably perturbed by environmental noise (see, e.g. [4]-[12]). Suppose that the parameter $r(t)$ is stochastically perturbed, with

$$r(t) \to r(t) + \sigma(t)\dot{B}(t),$$

where $\dot{B}(t)$ is white noise and $\sigma^2(t)$ stands for the intensity of the noise. Then this environmentally perturbed system can be described by the *Itô* equation

$$dx(t) = x(t)\left(1 - x^{\theta}(t)/K\right)\left[r(t)dt + \sigma(t)dB(t)\right], \tag{3}$$

and in the Stratonovič sense

$$dx(t) = r(t)x(t)\left(1 - x^{\theta}(t)/K\right)dt + \sigma(t)x(t)\left(1 - x^{\theta}(t)/K\right) \circ dB(t), \tag{4}$$

where $r(t)$ and $\sigma(t)$ are bounded and continuous functions on $[0, +\infty)$. Several authors have investigated the corresponding stochastic model ([5]-[11]).

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