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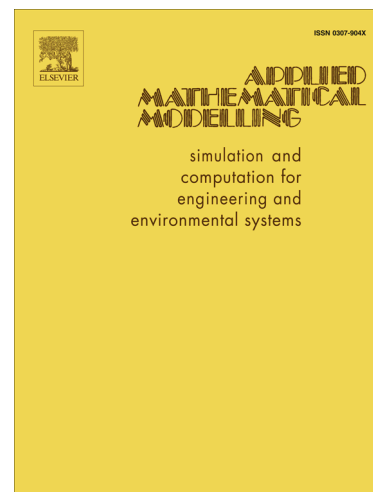
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## Stability of stochastic Richards growth model

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This paper is devoted to asymptotic analysis of equilibrium states of stochastic Richards equation. Both  $It\hat{o}$  and Stratonovič interpretations are investigated. Sufficient conditions for asymptotical stability of the zero solution and the positive equilibrium are established. Numerical simulations are introduced to illustrate the main results.

*Keywords:* Richards model, Brownian motion, Global asymptotical stability.

**1. Introduction**

Logistic model has been studied extensively owing to its theoretical and practical significance. The generalized logistic equation (Richards model[1, 2]) has the form

$$dx(t)/dt = rx(t) \left( 1 - x^\theta(t)/K \right), \quad (1)$$

where  $x(t)$  denotes the population size,  $r$  is the growth rate,  $\theta$  is a positive constant and  $K > 0$  is the carrying capacity. Obviously, if  $\theta = 1$ , model(1) is the classical logistic model. For the classical logistic model, a famous result is that if  $r < 0$  and  $0 \leq x_0 < \sqrt[\theta]{K}$ , then  $\lim_{t \rightarrow +\infty} x(t) = 0$ ; if  $r > 0$ , then  $\lim_{t \rightarrow +\infty} x(t) = K$  (see e.g. Murray [3]). As a matter of fact, the natural growth of many populations vary with  $t$ , for example, due to the seasonality. Therefore it is reasonable to consider the nonautonomous equation

$$dx(t)/dt = r(t)x(t) \left( 1 - x^\theta(t)/K \right), \quad (2)$$

where  $r(t)$  is bounded and continuous function on  $[0, +\infty)$ .

In the real world, population systems are inevitably perturbed by environmental noise (see, e.g. [4]-[12]). Suppose that the parameter  $r(t)$  is stochastically perturbed, with

$$r(t) \rightarrow r(t) + \sigma(t)\dot{B}(t),$$

where  $\dot{B}(t)$  is white noise and  $\sigma^2(t)$  stands for the intensity of the noise. Then this environmentally perturbed system can be described by the  $It\hat{o}$  equation

$$dx(t) = x(t) \left( 1 - x^\theta(t)/K \right) \left[ r(t)dt + \sigma(t)dB(t) \right], \quad (3)$$

and in the Stratonovič sense

$$dx(t) = r(t)x(t) \left( 1 - x^\theta(t)/K \right) dt + \sigma(t)x(t) \left( 1 - x^\theta(t)/K \right) \circ dB(t), \quad (4)$$

where  $r(t)$  and  $\sigma(t)$  are bounded and continuous functions on  $[0, +\infty)$ . Several authors have investigated the corresponding stochastic model ([5]-[11]).

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