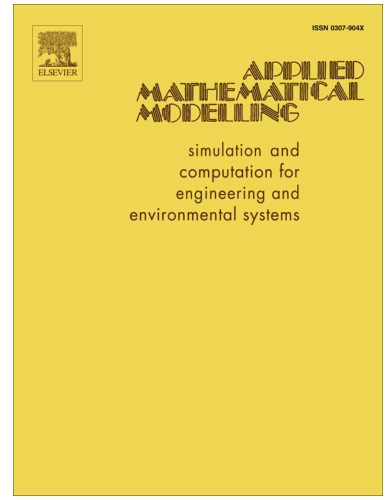

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A Penalty Decomposition Method for Rank Minimization Problem with Affine Constraints

Zheng-Fen Jin[a], Zhongping Wan[a*], Xiaoke Zhao[a] and Yunhai Xiao[b]

[a] School of Mathematics and Statistics, Wuhan University, Wuhan 430072, P. R. China.

[b] College of Mathematics and Information Science, Henan University, Kaifeng, 475004, P. R. China.

Abstract

The rank minimization problem with affine constraints is widely applied in the field of control, system identification, and machine learning, and attracted much attention and well studied in the past few years. Unlike most of the existing methods where a nuclear norm is used to approximate the rank term, in this paper, we apply the penalty decomposition method to solve the rank minimization problem directly. One subproblem can be solved effectively by using linear conjugate gradient method, and the other one has closed-form solutions by taking full use of the problem's favorable structure. Under some suitable assumptions, the convergence result for the proposed method is given. Finally, we do numerical experiments on randomly generated and real data, the results show that the proposed method is effective and promising.

Key words. rank minimization, penalty decomposition method, block coordinate descent method

AMS 2010 Subject Classification. 90C59, 90C99, 15A83

1. Introduction

In this paper, we consider the affine constrained rank minimization problem

$$\min_{X\in\mathbb{R}^{m\times n}} \operatorname{rank}(X), \quad \text{s.t. } \mathcal{A}(X)=b, \tag{1}$$

where $X\in\mathbb{R}^{m\times n}$ is a decision variable, $\mathcal{A}:\mathbb{R}^{m\times n}\to\mathbb{R}^p$ is a linear map and $b\in\mathbb{R}^p$ is a given measurement vector. This problem appears in many applications arising in various areas, for instance, the low-order realization of linear control system[1], system identification in engineering[2] and machine learning[3], etc. A particular case of problem (1) is the matrix completion problem

$$\min_{X\in\mathbb{R}^{m\times n}} \operatorname{rank}(X), \quad \text{s.t. } X_{i,j}=M_{i,j}, \quad \forall (i,j)\in\Omega, \tag{2}$$

where M is an unknown matrix with some available sampled entries, and Ω is a set of index pairs (i,j). Given a subset of entries of a matrix, problem (2) is to recover the missing entries to obtain a complete low-rank matrix. The problem (1) is generally viewed as NP-hard because of the combinational nature of

*Corresponding author. E-mail addresses: jinzhengfen@whu.edu.cn (Z.-F. Jin), mathwanzhp@whu.edu.cn (Z. Wan), zhaoxiaokehenan@126.com (X. Zhao), yhxiaomath@gmail.com (Y. Xiao) .