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Short communication

Parameter optimization of orthonormal basis functions for efficient rational approximations

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ABSTRACT

In this paper, the authors present an efficient procedure for optimal placement of poles in rational approximations by Müntz–Laguerre functions. The technique is formulated as the minimization of a quadratic criterion and the linear equations involved are efficiently expressed using the orthonormal basis functions. The presented technique has direct application in rational approximation and model order reduction of large-degree or infinite-dimensional systems.

1. Introduction

Rational orthogonal basis functions (OBF) are useful tools in the identification and modeling of linear dynamical systems and found numerous applications in control and signal processing [1]. In approximation problems using OBF, one of the major difficulties is the choice of the poles defining the functions. Due to their simplicity, Laguerre basis functions are often used. They have a real multiple-order pole whose choice is of great importance for computing low-order and good quality models. Much work has been done on the subject and optimal methods [2–5] or sub-optimal methods [6–8] have been proposed in literature. However Laguerre functions are poorly suited to compact modeling of systems possessing several time constants or resonant characteristics. Two-parameter Kautz functions are more adequate for modeling such systems but their efficiency is also limited. Techniques for an optimal or a suboptimal choice of the two-parameter Kautz poles are respectively presented in [9,10].

Finally for an effective approximation with a limited number of functions, the use of a more general orthogonal basis is preferable. Among them, Müntz–Laguerre basis functions that result from orthogonalization of a set of complex exponentials, have interesting properties [11]. Nevertheless few methods exist for properly choosing the poles in generalized OBF approximations. The conditions that the optimal poles of OBF models must satisfy have been investigated in [1,12] (in the discrete time case). These conditions are of great theoretical interest but generally cannot be solved in practical cases. On the other hand, asymptotically optimal pole locations aim to increase the convergence rate of the norm of the approximation error [1,13]. In this method, the minimization problem involves several independent variables and is faced with local minima and a cost function that is usually not differentiable. A simplistic approach consists in choosing the poles in accordance with the underlying dynamics of the system [14]. Such heuristics are rarely satisfactory in practice.

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In a modeling context, the optimal choice of Müntz–Laguerre parameters (poles) is closely related to the model order reduction (MOR) problem whose main objective is the computation of a low order denominator (or, in a state space representation, a low order state matrix) that can be used to efficiently approximate the original system. Most MOR methods require that the original model be in a rational form. The major interest in the use of Müntz–Laguerre functions is that one can deal with original systems described by rational or irrational transfer functions and also by physical measurements in both time and frequency domains. The present paper does not focus on the actual computation of the Müntz–Laguerre expansion but it is important to note that effective technics exist such as the ones described in [1,15].

In the following sections an original method of pole selection for the Müntz–Laguerre functions is presented. It is based on the construction of a family of functions related to the original transfer function and on the minimization of a modified quadratic error criterion. An efficient method for computing the required Gramian is also proposed. Gramians in general, have several useful properties [16] that have often attracted the interest of researchers working in various fields of system theory.

The article is organized as follows: Section 2 introduces the Müntz–Laguerre functions and describes the proposed procedure for parameters optimization. Section 3 details some properties of the procedure. Section 4 illustrates the performance of the method with a variety of numerical examples and comparisons with existing methods. Some demonstrations are available in the appendices.

2. Proposed procedure

2.1. Background

Let the Hardy space $\mathcal{H}_2$ consisting of all analytic and square-integrable functions in the open right half-plane with scalar product

$$\langle F, G\rangle = \frac{1}{2\pi}\int_{-\infty}^{+\infty} F(i\omega)\overline{G(i\omega)}d\omega, \tag{1}$$

and the norm $\|F\| = \sqrt{\langle F, F\rangle}$. In $\mathcal{H}_2$, Müntz–Laguerre bases are built by orthogonalization of complex exponential functions. They are defined in the Laplace domain by

$$\Phi_n(s) = \frac{\sqrt{2\mathcal{R}\{\alpha_n\}}}{s+\alpha_n}\prod_{l=1}^{n-1}\left(\frac{s-\bar{\alpha}_l}{s+\alpha_l}\right), \tag{2}$$

for $n = 1, 2, \ldots$, with $\mathcal{R}\{\alpha_l\} > 0\ \forall l$ where $\bar{\alpha}_l$ denotes the complex conjugate of α_l. Note that taking $\alpha_l = \alpha \in \mathcal{R}\ \forall l$ yields the 'single parameter' Laguerre functions. Moreover when α_l parameters are grouped by pairs of real or complex conjugate values, Müntz–Laguerre functions can be directly linked to Kautz functions [1].

Müntz–Laguerre functions are orthonormal, $\langle \Phi_n, \Phi_m\rangle = \delta_{n,m}$, and form a complete set in $\mathcal{H}_2$ under an easily satisfied condition [17,18]. It follows that any strictly proper transfer function $F(s)$ in the Hardy space $\mathcal{H}_2$ could be exactly represented with an infinite Müntz–Laguerre expansion as

$$F_\infty(s) = \sum_{n=1}^{\infty} d_n\Phi_n(s) = F(s), \tag{3}$$

where the expansion coefficients d_n are given by the inner products

$$d_n = \langle F, \Phi_n\rangle. \tag{4}$$

In practice series (3) is truncated

$$F_k(s) = \sum_{n=1}^{k} d_n\Phi_n(s), \tag{5}$$

where $\mathbf{d} = [d_1, d_2, \ldots, d_k]$ satisfy the optimality condition

$$\min_{\mathbf{d}}\|F - F_k\|. \tag{6}$$

The truncated series (5) defines a k-order rational approximation for $F(s)$ where the first Müntz–Laguerre parameters α_l $(l = 1, 2, \ldots, k)$ have a great impact on the quality of the model. Nevertheless their optimal choice, minimizing the quadratic error norm, is a nonlinear problem that usually cannot be solved in a simple way.

2.2. Model definition and identification

Due to the nonlinearity of the problem, the major difficulty in rational approximation is to take the 'best' choice for the poles of the model. The original idea developed in this paper is to consider the following expression