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Short communication

Global residue harmonic balance method for large-amplitude oscillations of a nonlinear system

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ABSTRACT

A new approach, namely global residue harmonic balance method, was used to deal with large-amplitude oscillations of a nonlinear system with inertia and static nonlinearities. Unlike other harmonic balance methods, all the former residual errors are introduced in the present approximation to improve the accuracy. Comparison of the result obtained using this approach with the exact one and existing results reveals that the high accuracy, simplicity and efficiency of the proposed procedure. The methods are valid for small as well as large amplitudes of oscillation, and can be easily extended to other strongly nonlinear oscillators.

1. Introduction

The problems related to large-amplitude oscillations of non-linear engineering structures have received considerable attention in the past years, where all the references cited in this section are only a small sample of the available literature on this topic [1–13]. There are several methods for approximating solutions for nonlinear problems with large parameters, such as harmonic based methods [5,6,13], coupled homotopy–variational formulation [7], variational approach [7,8], amplitude–frequency formulation [7,9], optimal homotopy asymptotic method [10,11] have been used to find approximate solutions to nonlinear problems.

In this paper, we put forward a novel approximate method, namely the global residue harmonic balance method, to determine the periodic solutions of free vibrations of cantilever beam. This oscillator is a conservative non-linear large-amplitude oscillatory system having inertia and static non-linearities. To obtain higher-order analytical approximations, all the residual errors are considered in the process of every order approximation. Excellent agreement of the approximate frequencies with the exact ones has been demonstrated and discussed. As can be seen, the results obtained in this paper revel that the method is very effective and convenient for conservative nonlinear oscillators.

2. Basic idea of global residue harmonic balance approach

For simplicity, we consider systems governed by equations of the form

$$\Phi(\ddot{u}, \dot{u}, u) = 0, \quad u(0) = A, \quad \dot{u}(0) = 0, \tag{1}$$

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where the over-dot denotes differentiation with respect to time t and A is the amplitude of the oscillations. For convenience, we assume Eq. (1) is a conservative system (i.e. $\Phi(-\ddot{u},-\dot{u},-u)=-\Phi(\ddot{u},\dot{u},u)$).

Eq. (1) describes a system oscillating with an unknown angular frequency ω. To determine the unknown frequency, we introduce a new independent variable $\tau=\omega t$. Then Eq. (1) becomes

$$\Phi(\omega^2 u'',\omega u',u)=0,\quad u(0)=A,\ u'(0)=0, \tag{2}$$

where prime denotes the derivative with respect to τ.

Considering the periodic solution does exist, it may be better to approximate the solution $u(\tau)$ by such a set of base functions

$$\{cos((2k-1)\tau)|\ k=1,2,3,\ldots\}. \tag{3}$$

According to Eq. (3), the initial approximate periodic solution satisfying initial conditions in Eq. (2) is

$$u_{(0)}(\tau)=Acos(\tau),\quad \tau=\omega_{(0)}t, \tag{4}$$

where $\omega_{(0)}$ is an unknown constant to be determined later.

To improve the accuracy, we will use the residual of the initial approximation. Substituting Eq. (4) into Eq. (2), we obtain the initial residual

$$R_0(\tau)=\Phi\left(\omega_{(0)}^2 u''_{(0)},\omega_{(0)}u'_{(0)},u_{(0)}\right). \tag{5}$$

If $R_0(\tau)=0$, then $u_0(\tau)$ happens to be the exact solution. Generally, such case will not arise for nonlinear problems.

Eq. (5) should not contain secular terms of $cos(\tau)$. Equating its coefficients to zero, we can determine the unknown constant $\omega_{(0)}$ and taking it as the approximation ω_0. Then, the zero-order approximation u_0 is also obtained of the form

$$u_0(\tau)=Acos(\tau),\quad \tau=\omega_0 t, \tag{6}$$

This yields the initial residual

$$R_0(\tau)=\Phi(\omega_0^2 u_0'',\omega_0 u_0',u_0). \tag{7}$$

In the following, we consider an iterative method by expanding $u(\tau)$ in a series with respect to the embedding parameter p of the form

$$u(\tau)=u_{(k-1)}(\tau)+pu_k(\tau),\quad \omega^2=\omega_{(k-1)}^2+p\omega_k,\quad k=1,2,3,\ldots, \tag{8}$$

where

$$\begin{aligned} &u_{(k-1)}(\tau)=u_{(k-2)}(\tau)+u_{k-1}(\tau),\quad \omega_{(k-1)}^2=\omega_{(k-2)}^2+\omega_{k-1},\\ &u_k(\tau)=\sum_{i=1}^{k}a_{2i+1,k}(cos(\tau)-cos((2i+1)\tau)),\quad k=2,3,\ldots, \end{aligned} \tag{9}$$

where p is the order parameter with values in the interval $[0,1]$, and the kth-order approximate solutions of $u(\tau)$ and ω can be obtained by taking $p=1$.

Given the zero-order approximation equation (6) and the residual equation (7), then the first-order approximate periodic solution and frequency can be written as

$$u(\tau)=u_0(\tau)+pu_1(\tau),\quad \omega^2=\omega_0^2+p\omega_1. \tag{10}$$

Substituting Eq. (6) into Eq. (2) and collecting the coefficients of p, we can get

$$F_1(\tau,\omega_1,u_1(\tau))\triangleq\left(\omega_1\frac{\partial}{\partial(\omega^2)}+u_1''\frac{\partial}{\partial u''}+u_1'\frac{\partial}{\partial u'}+u_1\frac{\partial}{\partial u}\right)\Phi_0, \tag{11}$$

where $\partial\Phi_0/\partial u$ denotes that $\partial\Phi/\partial u$ is to be evaluated at the zero-order approximation after differentiation etc. It is noted that Eq. (11) is linear with respect to ω_1 and u_1.

Considering the solution has the form of Eq. (3), we choose

$$u_1(\tau)=a_{3,1}(cos(\tau)-cos(3\tau)). \tag{12}$$

Substituting Eq. (12) into Eq. (11), we consider the following equation

$$F_1(\tau,\omega_1,u_1(\tau))+R_0(\tau)=0. \tag{13}$$

In this way, all the residual errors of the zero-order approximation $R_0(\tau)$ are introduced into Eq. (13) which would improve the accuracy.

The left hand side of Eq. (13) should not contain the terms $cos(\tau)$ and $cos(3\tau)$ based on Galerkin technique. Letting their coefficients be zeros, we obtain two linear equations containing two unknowns ω_1 and $a_{3,1}$. Then the two unknown constants can be solved easily. Thus, we get the first-order approximation

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