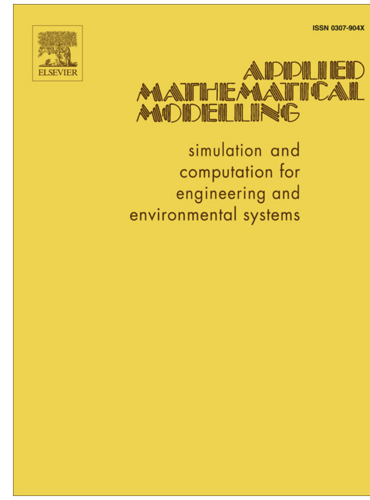
Accepted Manuscript



A Bessel Collocation Method for Solving Fractional Optimal Control Problems

Emran Tohidi, Hassan Saberi Nik

PII: S0307-904X(14)00306-0
DOI: http://dx.doi.org/10.1016/j.apm.2014.06.003
Reference: APM 10051

To appear in: *Appl. Math. Modelling*

Please cite this article as: E. Tohidi, H.S. Nik, A Bessel Collocation Method for Solving Fractional Optimal Control Problems, *Appl. Math. Modelling* (2014), doi: http://dx.doi.org/10.1016/j.apm.2014.06.003

A Bessel Collocation Method for Solving Fractional Optimal Control Problems

Emran Tohidi[†,*], **Hassan Saberi Nik**[‡]

† Department of Mathematics, Islamic Azad University, Zahedan Branch, Zahedan, Iran.
‡ Department of Mathematics, Islamic Azad University, Neyshabour Branch, Neyshabour, Iran.
*Corresponding author: emrantohidi@gmail.com

Abstract

In the present paper, we apply the truncated Bessel series approximation by using collocation scheme, for solving linear and nonlinear fractional optimal control problems (OCPs) indirectly. Therefore, the necessary (and also sufficient in most cases) optimality conditions are stated in a form of nonlinear (or linear) fractional two-point boundary value problem (TPBVP). For solving this mentioned TPBVP, we generalize a new numerical method (which is called the Bessel collocation method). One of the best advantages of this generalization is that, there is no need to use operational matrices of differentiation and also the new generalized idea can be implemented in any mathematical software. Some numerical examples are provided to confirm the accuracy of the proposed method. All of the numerical computations have been performed on a PC using several programs written in MAPLE 13.

Keywords: Fractional optimal control problems, truncated Bessel series, Collocation Method.

1 Introduction

Control problems for systems governed by ordinary (or partial) differential equations arise in many applications, e.g., in astronautics, aeronautics, robotics, and economics [1,2]. Experimental studies of such problems go back recent years and computational approaches have been applied since the advent of computer age. The solution of practical control systems usually has special difficulties. Moreover, In classical theory of control, just input-output signals are considered and the basic deficiency of this theory is that it is only applicable for time invariant linear systems. Therefore, presentation of an applicable numerical approach for solving practical control problems has considerable importance.

We recall that, the approaches for numerical solutions of optimal control problems (OCPs) may be divided into two major classes: indirect methods and the direct methods. The indirect methods are based on the Pontryagin maximum principle (PMP) and require the numerical solution of boundary value problems that result from the necessary conditions of optimal control [16]. Direct optimization methods transcribe the (infinite-dimensional) continuous problem to a finite-dimensional nonlinear programming problem (NLP) through some parametrization of the state and/or control variables. In the direct methods, initial guesses have to be provided only for physically intuitive quantities such as the states and possibly controls. The indirect schemes are based on optimizing then discretizing the main OCPs, meanwhile the direct methods are based on discretizing then optimizing the main OCPs. One of the best properties of the indirect schemes is the high credit of the obtained approximate solution of the main OCPs. This specific property is based on satisfying the first order of necessary conditions that originated from the calculus of variation and the PMP. In this paper, after imposing PMP to the considered fractional OCPs, we obtain a fractional two-point boundary value problem (TPBVP) such that for solving this equation we generalize a new collocation method which is based on the truncated Bessel series [27]. It should be mentioned that, the Bessel collocation method (BCM) which was used in the works [28–31] just can be applicable for solving linear and some special nonlinear cases (in polynomial forms), because in these works the operational matrix of differentiation has a basic role. Therefore, a generalization of this idea should be constructed, since in most case studies of real