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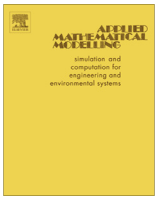

Wave modes of a pre-stressed thick tube conveying blood on the viscoelastic foundation

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ABSTRACT

Wave propagation of an artery is a fluid–structure interaction problem. It is very complicate. Therefore, the conventional theories for circulation of arteries are emphasized on fluid behavior, some simplified models for experimental utility or the thin-walled tube theory. Based on the geometry of an artery, the thick-walled tube theory is reasonable. In this study, a new mathematical model is proposed to describe the wave propagation through the isotropic elastic thick tube filled with viscous and incompressible fluid. Moreover, the tube is supported by the elastic muscle and simulated as the viscoelastic foundation. The radial, axial and flexural vibrations of a tube wall are introduced simultaneously. These wave modes are generally called as the flexural, Young and Lamb modes. In the literatures according to different assumptions, the Young and Lamb modes were independently derived and independent to the wave frequency. Because these conventional models are over simplified, the corresponding investigations are incomplete and inaccurate. Moreover, the present thick-walled tube theory is compared with the thin-walled tube theory and these conventional limiting theories. The dispersion curves and the energy transmissions of the three modes are investigated. It is illustrate that the energy transmitted through the artery tube is consistent to the experiment. Moreover, it is found that the effects of the viscoelastic foundation constants on the wave speed and the transmission is significant. When the foundation constant is large enough, some corresponding mode will disappear.

1. Introduction

No matter animals or human beings, the blood circulation plays an important role in maintaining body working well. Many scientists investigated the heart vascular system, and proposed several theories and models to explain the performances of the blood circulation system. In general, the ratio of tube thickness to the tube radius of the artery of man or dog is from 0.06 to 0.13 [1]. The thick-walled tube theory should be considered for simulating the behavior of the artery. However, since the calculations appeared to be very laborious, the thin-walled tube theory was often introduced [2,3].

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Nomenclature

A	surface area of the fluid element
c_{ij}	elastic constants
C_r, C_x	damping coefficients in the r- and x-directions, respectively
C_w	wave speed
D	diameter of the tube
E	Young's modulus
f	friction factor of the tube wall
G	shear modulus
h	thickness of the tube wall
k	wave number
K_r, K_x	elastic foundation constants in the r- and x-directions, respectively
L	tube length
M_{ij}	twisting moment perpendicular to i-plane along j-direction or perpendicular to j-plane along i-direction per unit length
M_x	bending moment perpendicular to x-plane along θ-direction per unit length
M_θ	bending moment perpendicular to θ-plane along x-direction per unit length
N_{ij}	shearing force perpendicular to i-plane along j-direction or perpendicular to j-plane along i-direction per unit length
N_x	normal force perpendicular to x-plane along θ-direction per unit length
N_θ	normal force perpendicular to θ-plane along x-direction per unit length
$\hat{n}$	normal direction of surface
p	liquid pressure at the wall
R	average radius of tube
Re	Reynold's number
R_i	inner radius of tube
R_o	outer radius of tube
r_{et}	percentage of energy propagations through the tube
t	time variable
u_r, u_θ, u_x	total displacements in r-direction, in θ-direction and in x-direction
u	displacement in x-direction
w	displacement in r-direction
$\vec{v}$	the flow velocity
V	volume of the fluid element

Greek symbols

β	angle due to bending
γ_{ij}	shearing strain on i–j plane
ε	normal strain
η	flow velocity distribution
υ	Poisson's ratio
ρ	density
σ	normal stress
τ_{ij}	shearing stress perpendicular to i-plane along j-direction
ω	wave frequency

Superscript, subscripts

–	amplitude
max	maximum quantity
r	radial coordinate
θ	transverse coordinate
x	axial coordinate
f	fluid
s	solid
0	static quantity due to pre-stressed
1	disturbed quantity due to wave propagation

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