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Short communication

Modelling and stabilization of a nonlinear hybrid system of elasticity

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ABSTRACT

In this article, we briefly present a model which consists of a non-homogeneous flexible beam clamped at its left end to a rigid disk and free at the right end, where another rigid body is attached. We assume that the disk rotates with a non-uniform angular velocity while the beam is supposed to rotate with the disk in another plane perpendicular to that of the disk. Thereafter, we propose a wide class of feedback laws depending on the assumptions made on the physical parameters. In each case, we show that whenever the angular velocity is not exceeding a certain upper bound, the beam vibrations decay exponentially to zero and the disk rotates with a desired angular velocity.

1. Introduction

This work is concerned with a non-homogeneous elastic beam clamped to the center of a disk and free at the other end where a body with mass m is attached. Such systems arise in the study of large scale flexible space structures. The disk is supposed to rotate around the x-axis (see Fig. 1) without friction. In turn, the beam is clamped at the left-end $x=0$, constrained to the x–y plane and all the deflections are assumed to be parallel to the y-axis (see Fig. 1). Consequently, it follows from [1] that

$$\begin{cases} \rho(x)y_{tt} + (EI(x)y_{xx})_{xx} = \rho(x)\,\omega^2(t)y, & (x,t)\in(0,\ell)\times(0,\infty),\\ y(0,t)=y_x(0,t)=0, & t>0,\\ \frac{d}{dt}\left\{\omega(t)\left(I_d+\int_0^\ell \rho(x)y^2(x,t)\,dx\right)\right\} = \beta\mathcal{T}(t), & t>0. \end{cases} \tag{1.1}$$

Here x denotes the position and t represents the time. Moreover, y is the beam's displacement, ω is the angular velocity of the disk, ℓ is the length of the beam, I_d is the disk's moment of inertia and $EI(x),\ \rho(x)$ are respectively the flexural rigidity and the mass per unit length of the beam satisfying

$$0<\rho_0<\rho(x)\in C^4[0,\ell],\quad 0<EI_0<EI(x)\in C^4[0,\ell]. \tag{1.2}$$

Furthermore, β is a positive feedback gain and $\mathcal{T}(t)$ is the control torque.

Now, we turn to the dynamics of the rigid body attached to the other end of the beam. We have [2]:

$$y(\ell,t)=\varsigma(t),\quad y_x(\ell,t)=\vartheta(t),$$

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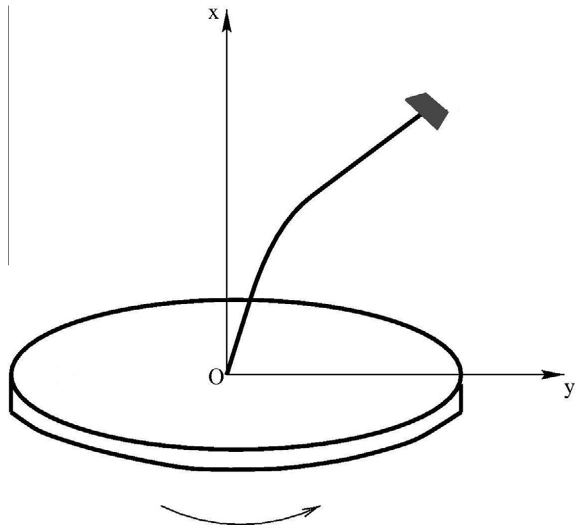

Fig. 1. Disk-beam-body system.

in which $\varsigma(t)$ is the transverse displacement of the centroid of this rigid body, while $\vartheta(t)$ gives the direction its normal makes with the x-axis. Next, we shall neglect, as in [2], the effect of the non-inertial force terms on the object of mass m. Then based on the Newton–Euler principles, the dynamics of the rigid body of mass m are given by (see [2] for more details)

$$\begin{cases} m\ddot{\varsigma}(t) = my_{tt}(\ell,t) = (EI(x)y_{xx})_x(\ell,t) + \alpha_1\Theta_1(t), & t>0,\\ J\ddot{\vartheta}(t) = Jy_{xtt}(\ell,t) = -(EI(x)y_{xx})(\ell,t) + \alpha_2\Theta_2(t), & t>0, \end{cases} \tag{1.3}$$

where J is the moment of inertia of the rigid body attached at the right end of the beam; α_1 and α_2 are nonnegative constant feedback gains such that $\alpha_1 + \alpha_2 \neq \mathbf{0}$ and $\Theta_1(t)$, $\Theta_2(t)$ are respectively the control force and the control moment. This, together with (1.1) and (1.3), allows us to claim that the dynamics of motion of our global system are given by the following system

$$\begin{cases} \rho(x)y_{tt} + (EI(x)y_{xx})_{xx} = \rho(x)\,\omega^2(t)y, & (x,t)\in(0,\ell)\times(0,\infty),\\ y(0,t) = y_x(0,t) = 0, & t>0,\\ my_{tt}(\ell,t) - (EI(x)y_{xx})_x(\ell,t) = \alpha_1\Theta_1(t), & t>0,\\ Jy_{xtt}(\ell,t) + (EI(x)y_{xx})(\ell,t) = \alpha_2\Theta_2(t), & t>0,\\ \frac{d}{dt}\left\{\omega(t)\left(I_d + \int_0^\ell \rho(x)y^2(x,t)\,dx\right)\right\} = \beta\mathcal{T}(t), & t>0. \end{cases} \tag{1.4}$$

In this work, we shall provide several feedback control laws and then show the exponential stability of the closed loop system. To be more precise, the stabilization result will be established in a number of situations depending on the smallness of the dynamical terms $my_{tt}(\ell,t)$ and $Jy_{xtt}(\ell,t)$. This extends the results available in literature in two directions. First, our results generalize those of [3–9] where neither the acceleration term $my_{tt}(\ell,t)$ nor the moment of inertia term $Jy_{xtt}(\ell,t)$ is present in the system (1.4). Secondly, we are also able to extend the stability results of [10–12,2] and related works to the case of the presence of a nonlinear coupling term $\rho(x)\omega^2(t)y(x,t)$. The crucial tool of the proof of our main results, namely, the exponential stability of the closed loop system is the utilization of the principal theorem in [13] for an uncoupled system. This strategy, due to Laousy et al. [7], has been adopted in many previous works [3–6].

The paper is organized as follows. In the next section, we shall assume that $my_{tt}(\ell,t)$ is not neglected and accordingly, we provide two different feedback laws depending on the smallness of the other dynamical term $Jy_{xtt}(\ell,t)$. It is worth mentioning that, in this case, the main difference of the controls resides in the simplicity feature. In fact, the first controls are of higher order whereas the second ones are known to be simple. Section 3 will be devoted to a thorough analysis of the system (1.4) without the acceleration term $my_{tt}(\ell,t)$. Once again, in such a situation, two feedback laws are proposed to stabilize the system. Finally, this note closes with conclusions and discussions.

2. The acceleration term $my_{tt}(\ell,t)$ is not negligible

Throughout this section, it will be assumed that the dynamical term $my_{tt}(\ell,t)$ cannot be neglected. Then, we will deal with the stabilization problem of the system (1.4) when the other dynamical term $Jy_{xtt}(\ell,t)$ has a significant value as well as when it is too small to be taken into consideration.