ARTICLE IN PRESS

Applied Mathematical Modelling xxx (2014) xxx–xxx

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Optimal pricing and ordering policies for non-instantaneously deteriorating items under order-size-dependent delay in payments

Chun-Tao Chang [a,*], Mei-Chuan Cheng [b], Liang-Yuh Ouyang [c]

[a] *Department of Statistics, Tamkang University, Tamsui, Taipei 25137, Taiwan, ROC*
[b] *Department of International Business, Hsin Sheng College of Medical Care and Management, Longtan, Taoyuan 32544, Taiwan, ROC*
[c] *Department of Management Sciences, Tamkang University, Tamsui, Taipei 25137, Taiwan, ROC*

ARTICLE INFO

Article history:
Received 4 October 2011
Received in revised form 29 May 2014
Accepted 8 July 2014
Available online xxxx

Keywords:
Inventory
Non-instantaneous deteriorating items
Pricing
Trade credit

ABSTRACT

In today's competitive business transactions, the supplier may permit his/her retailers a delay in payment in order to encourage the retailers to buy more. During the permissible delay period, the retailer is allowed to postpone paying for the products bought without incurring any interest. In this study, we consider an inventory system with non-instantaneously deteriorating items in circumstances where the supplier provides the retailer with various trade credits linked to order quantity. First, we develop a mathematical model to identify the optimal pricing and ordering policies for maximizing the retailer's total profit. This followed by a discussion of the characteristics of the optimal solution. We then propose some algorithms for finding the optimal solutions. Finally, numerical examples are presented and a sensitivity analysis is undertaken to illustrate the proposed model.

1. Introduction

Researchers assume, in the classical inventory model, that the value of inventory items is unaffected by the duration of time. In practice, however, many items deteriorate during the normal storage period. Chemicals, volatile liquids, blood stored in blood banks, and electronic components deteriorate significantly. Deterioration is defined as the decay, damage, spoilage, evaporation, or drying out of products. Thus, the ideal case envisioned by the classical model remains an ideal one. The effects of deterioration are significant in many inventory systems, making the problem of how to control and maintain inventories of deteriorating items a major issue for decision makers in modern organizations. In addressing this issue, Ghare and Schrader [1] first proposed a model for an exponentially decaying inventory, which Covert and Philip [2] extended to a two-parameter Weibull distribution. Goyal and Giri [3] classified the previous studies and provided a detailed review of the literature on deteriorating inventory. Jaber et al. [4] developed a mathematical model that determines batch sizes for deteriorating items while minimizing entropy. Other interesting articles that cover the topic include Shah and Jaiswal [5], Aggarwal [6], Dave and Patel [7], Sachan [8], Hariga [9], Skouri and Papachristos [10], Chang [11], Liao [12] and Jaber et al. [13].

In the existing literature, all the models assume that the deterioration of items in an inventory starts from the moment of their arrival in stock. However, in real life there is a time span during which most commodities maintain their quality or

* Corresponding author. Fax: +886 2 26209732.
E-mail addresses: chuntao@stat.tku.edu.tw (C.-T. Chang), mccheng@hsc.edu.tw (M.-C. Cheng), liangyuh@mail.tku.edu.tw (L.-Y. Ouyang).

http://dx.doi.org/10.1016/j.apm.2014.07.002
0307-904X/

original condition, that is, during which no deterioration occurs. Beyond this period, however, some of the items will start to decay. Wu et al. [14] defined this phenomenon as "non-instantaneous deterioration." It exists commonly among medicines, firsthand vegetables, and fruits, all of which can maintain their fresh quality for a short span of time. During this initial time period, there is almost no spoilage. For these kinds of items, the assumption that the deterioration begins to occur as soon as the retailer receives the items may cause retailers to adopt inappropriate replenishment policies as a result of overvaluing the total relevant inventory cost. Chang et al. [15] proposed optimal replenishment policies for non-instantaneously deteriorating items with stock-dependent demand. Their model set a maximum inventory level to reflect the limited shelf space of most retail outlets. Yang et al. [16] developed a model in which shortages are accepted and partially backlogged with a variable backlogging rate dependent on the waiting time for the next replenishment. Geetha and Uthayakumar [17] developed an economic order quantity (EOQ) model for non-instantaneously deteriorating items with permissible delay in payments in which model shortages are allowed and partially backlogged. Maihami and Kamalabadi [18] presented a joint pricing and inventory model for non-instantaneously deteriorating items with a price-and-time-dependent demand function. Our study demonstrates the importance of taking into consideration the inventory problems associated with non-instantaneously deteriorating items in the inventory management system.

In addition to the inventory problem, this article addresses the issue of payment to suppliers. The traditional EOQ model tacitly assumes that payment must be made to the supplier immediately after retailers receive the items. In reality, suppliers, hoping to promote the sale of their products, are willing to offer retailers a payment delay period, known as a trade credit period. The trade credit is the largest use of capital for the majority of business-to-business (B2B) and business-to-customer (B2C) sellers and is a critical source of capital for a majority of all businesses. This is an important and popular topic because it characterizes the real situation in the market. During the trade credit period, retailers can actually gain interest from non-payment and sales income, while the supplier loses interest income during this period. Thus, the delay in payment to suppliers serves as a kind of price discount. Because paying later indirectly reduces the purchase cost, retailers are motivated to increase their order quantity.

Issues related to trade credits have been considered by several researchers. Goyal [19] first developed the EOQ model with a permissible delay in payment to determine the optimal order quantity. Aggarwal and Jaggi [20] extended Goyal's model to allow for deteriorating items, which Jamal et al. [21] generalized to allow for shortages. Hwang and Shinn [22] considered demand, which is a function of retail price, and developed the optimal pricing and lot-sizing policy for the retailer in the case of a permissible delay in payments. Teng [23] modified Goyal's model, assuming that the selling price is not equal to the purchasing price, to find that it is economically viable for a well-established buyer to frequently order a lower quantity and take advantage of the benefits of a permissible delay. Teng et al. [24] combined the approaches of Hwang and Shinn [22] and Teng [23], and presented a pricing and lot-sizing model for retailers in which the supplier provides a permissible delay in payments. Urban [25] proposed an extension of inventory models, incorporating financing agreements with both suppliers and customers. Ouyang et al. [26] developed an inventory model for non-instantaneous deteriorating items with a permissible delay in payments. Based on this model, they provided theorems that characterize the optimal solution and a straightforward method for finding the optimal replenishment cycle time and order quantity under various circumstances. Some other studies of permissible delay in payments are Davis and Gaither [27], Arcelus and Srinivasan [28], Shah [29], Khouja and Mehrez [30], Sarker et al. [31], Chang and Wu [32], Chang [33], Ouyang et al. [34,35], Chang et al. [36], Ho, et al. [37], Sana and Chaudhuri [38], Chang et al. [39], Teng and Chang [40], Chen and Kang [41], Liang and Zhou [42], Roy and Samanta [43], Jaber [44], Lou and Wang [45], and Jaber and Osman [46].

Most of the earlier studies dealing with inventory problems in circumstances of permissible delay in payments discuss a case in which the delay in payments is independent of the quantity ordered. However, in today's business transactions, in order to encourage the retailer to order large quantities, the supplier may offer a permissible delay of payment for large quantities but require immediate payment for small quantities. Hence, the supplier may set a predetermined order quantity below which delay in payment is not permitted and payments must be made immediately. For order quantities above this threshold, the trade credit period is permitted. Khouja and Mehrez [29] investigated the effect of supplier credit policies on the optimal order quantity. They addressed two types of supplier credit policies: the first type is one in which credit terms are independent of the quantity ordered, and the second type is one in which the credit terms are linked to the order quantity. Shinn and Hwang [47] analyzed the problem of the retailer who has to decide his/her sale price and order quantity simultaneously in the case of an order-size-dependent delay in payments. Chang et al. [48] developed an EOQ model with deteriorating items where suppliers link credit to order quantity. Chung and Liao [49] discussed the optimal replenishment cycle time for an exponentially deteriorating product under the condition that the delay in payments depends on the quantity ordered. Other researchers who address this topic include Chang [32], Chung et al. [50], Liao [51], Ouyang et al. [52,53], Chang et al. [54], and Yang et al. [55].

In a competitive market, suppliers may offer different trade credit periods with different predetermined quantities to increase retailers' choices and encourage retailers to order higher quantities. Hence, in this article, we will develop an appropriate inventory model for non-instantaneously deteriorating items where suppliers provide a permissible payment delay schedule linked to order quantity. The rest of the article is organized as follows. The assumptions and notations used in this study are presented in Section 2. In Section 3, a mathematical model is developed to show the pricing and ordering policies that will maximize profits in various trade credit situations. We then discuss the necessary and sufficient conditions for an optimal solution and develop the solution algorithms. In Section 4, numerical examples are provided to illustrate the