ARTICLE IN PRESS

Applied Mathematical Modelling xxx (2014) xxx–xxx

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Efficient structural reliability analysis method based on advanced Kriging model

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ARTICLE INFO

Article history:
Received 3 June 2013
Received in revised form 25 January 2014
Accepted 3 July 2014
Available online xxxx

Keywords:
Reliability analysis
Kriging model
Most probable region
Limit state function
Failure probability

ABSTRACT

Reliability analysis becomes increasingly complex when facing the complicated expensive-to-evaluate engineering applications, especially problems involve the implicit finite element models. In order to balance the accuracy and efficiency of implementing reliability analysis, an advanced Kriging method is proposed for efficiently analyzing the structural reliability. The method starts with an incipient Kriging model built from a very small number of samples generated by the simple random sampling method, then determines the most probable region in the probabilistic viewpoint and chooses the subsequent samples located in this region by employing the probabilistic classification function. Besides, the leave-one-out technique is used to update the current model. By locating samples in the probabilistic most probable region, only a small number of samples are used to build a precise surrogate model in the end, and only a few actual limit state function evaluations are required correspondingly. After the high quality surrogate of the implicit limit state is available by the advanced Kriging model, the Monte Carlo simulation method is employed to implement reliability analysis. Some engineering examples are introduced to demonstrate the accuracy and efficiency of the proposed method.

1. Introduction

Reliability analysis aims at evaluating the safety level of systems or structures. In the past few decades, many reliability analysis techniques have been developed. Difficulty in computing the failure probability has lead to the development of various approximation methods [1], among which the first-order reliability method (FORM) [2,3] and the second-order reliability method (SORM) [4–6] focus on searching for a single most probable point (MPP) in the failure domain, and then quantify the reliability by building a low-order approximation to the limit state function at MPP. These methods may wrongly assess the safety level in case of multiple MPPs, besides, they rely on MPP convergence and the evaluated results are affected by the precision of the limit state function approximation to a great extent.

As presented by many practitioners, engineering applications are always complex with highly nonlinear limit state models. Thus engineers resort to study the sampling methods, which do not rely on a lower-order approximation of the limit state function. The Monte Carlo simulation (MCS) technique [7–9] is a basic reference approach and is widely used. However, for the implicit limit state models where the finite element model (FEM) analysis is employed to obtain the output, the MCS

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http://dx.doi.org/10.1016/j.apm.2014.07.008

Please cite this article in press as: L. Zhang et al., Efficient structural reliability analysis method based on advanced Kriging model, Appl. Math. Modell. (2014), http://dx.doi.org/10.1016/j.apm.2014.07.008

method is infeasible due to the large computational cost. Based on MCS, the importance sampling (IS) method [10,11] is developed. Melchers [10] employed a standard normal probability density function (PDF) centered on the MPP. Au [11] used a kernel smoothing approximation of the optimal instrumental PDF built from some failed samples. IS method requires fewer evaluations of the actual limit state function comparing with MCS, however it needs large number of evaluations all the same with respect to the rare events.

In order to reduce the calls of limit state functions, especially for the FEM analysis, some approximation methods based on the meta-models are proposed including quadratic response surfaces [12–14], neural networks [15], support vector machines [16,17] and Kriging [18–21]. However, it is often difficult to decide how many samples should be selected to construct the surrogates and it is difficult to quantify the error of the surrogate model. The traditional Kriging based methods used a number of randomly selected samples to build the surrogate, and the accuracy of the approximate model depends on the information provided by the given samples. If few samples are used, the prediction capability of the approximate model would be insufficient. On the contrary, if large numbers of samples are used, the accuracy can be ensured, but the correspondingly computational cost would be expensive, especially for the computational intensive models.

Obviously, an efficient meta-model based reliability analysis method is needed to balance the accuracy and cost. Bichon [21,22] proposed an expected feasibility function based on the Kriging model and depended on it to locate the samples near the limit state, which decreased the actual limit state function evaluations. Dubourg [23,24] employed a probabilistic classification function based on the Kriging model to approximate the failure indicator in order to refine the models and build a quasi-optimal importance sampling density. This paper starts from the probabilistic viewpoint and proposes an advanced Kriging-based method to efficiently estimate the reliability of the structural system. The method begins with an initial Kriging model constructed from a very small number of samples obtained by the simple random sampling method, then employs the probabilistic classification function to determine the most probable region, and selects the subsequent samples with high level of uncertainty to enrich the experiment points for updating the model. Besides, the leave-one-out technique is used as the stopping criterion to refine the model. By choosing the subsequent samples which locate in the most probable region with the probabilistic viewpoint, only a small number of evaluations of the actual limit state function are needed to build an accurate meta-model.

This paper is organized as follows. Section 2 reviews the basic reliability analysis and the Kriging method. Section 3 presents methods to find the probabilistic most probable region and choose the subsequent experiment samples in this region for refining the model. Section 4 gives the implementation progress of reliability analysis by the proposed advanced Kriging method. Section 5 illustrates the accuracy and efficiency of the proposed method. Section 6 provides the conclusions.

2. Reliability analysis

2.1. Basic reliability methods

The goal of reliability analysis is to compute the failure probability P_f to evaluate the safety level of systems or structures. For the response function $Z = g(x)$ relating with the n-dimensional independent random input vector $x = \{x_1, x_2, \ldots, x_n\}$, the failure probability P_f is defined by

$$P_f = \int_F f_x(\boldsymbol{x})d\boldsymbol{x} = \int_{R^n} I_{g\leqslant 0}(\boldsymbol{x})f_x(\boldsymbol{x})d\boldsymbol{x}, \tag{1}$$

where $f_x(\boldsymbol{x}) = \prod_{i=1}^n f_{x_i}(x_i)$ is the joint probability density function (PDF) of random variable vector $\boldsymbol{x}$, $f_{X_i}(x_i)$ is the marginal PDF of x_i. The integration is performed over the failure region F, which is defined by the response function $Z = g(\boldsymbol{x})$ as $F = \{\boldsymbol{x} : g(\boldsymbol{x}) \leqslant 0\}$. $I_{g\leqslant 0}(x)$ is the failure indictor, it equals to one if $g(\boldsymbol{x}) > 0$ and zero otherwise.

The basic approximation to compute the failure probability is FORM, which linearizes the limit state surface and computes the failure probability by

$$P_f \approx \Phi(-\beta), \tag{2}$$

where β is the reliability index and represents the distance from the origin to the MPP in the standard normal space. SORM computes the failure probability in the same way by using a quadratic surface fitted at the MPP. However, for a complex engineering application, the limit state functions may be multimodal and possess multiple MPPs, thus the reliability estimated by the methods only on the information of the single MPP may be inaccurate.

Generally, the MCS method is a basic simulation and the results are used as references. It is a simulation technique. The MCS estimator is then derived as

$$\widehat{P}_{f\ MC} = E_x[I_{g\leqslant 0}(\boldsymbol{x})] = \frac{1}{N}\sum_{i=1}^{N} I_{g\leqslant 0}(\boldsymbol{x}^{(k)}), \tag{3}$$

where $x^{(k)}(k = 1, \ldots, N)$ are a set of random input samples and $E[\cdot]$ is the expectation operator. According to the central limit theorem, this estimator is unbiased. For the events with very rare failure probability, in order to obtain a convergent result, N should be large enough (generally, $N = (10^2 \sim 10^4)/P_f$). Note that it is efficient to implement MCS estimation for the problem with explicit limit state function. However, engineering problems are often characterized by an implicit input–output