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Primal mixed solution to unconfined seepage flow in porous media with numerical manifold method ☆

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ABSTRACT

The major difficulty in the analysis of unconfined flow in porous media is that the free surface is unknown *a priori*, where the nonlinearity is even stronger than the unsaturated seepage analysis. There is much space for both the adaptive mesh methods and the fixed mesh methods to improve. In this study, firstly two variational principles fitted to the numerical manifold method (NMM) are formulated, each of which enforces the boundary conditions and the material interface continuity conditions. In the setting of the NMM together with the moving least squares (MLS) interpolation, then the discretization models corresponding to the variational formulations are built, which are utilized to locate the free surface and scrutinize the computational results respectively. Meanwhile, a novel approach is developed to update the free surface in iteration. With high accuracy and numerical stability but no need to remesh, the proposed procedure is able to accommodate complicated dam configuration and strong non-homogeneity, where internal seepage faces may develop, a seldom touched problem in the literature.

1. Introduction

Due to the importance in engineering and strong nonlinearity in mathematics, problems of seepage with free surfaces, called unconfined seepage problems, are paid extensive attention from both engineers and mathematicians. The unconfined seepage analysis belongs to geometrical nonlinearity, while the unsaturated seepage analysis belongs to material nonlinearity and needs more soil parameters or curves, some of which are hard to obtain, such as soil–water characteristic curves. But the former's nonlinearity is stronger than the latter, because usually the analysis domain in the unsaturated seepage analysis does not change in iteration.

The procedures developed for the unconfined seepage problems are diverse, including the adaptive mesh methods and the fixed mesh methods. The adaptive mesh methods are easy to understand in concept, but they usually have to remesh explicitly during iteration, leading to laborious mesh generation or adjustment. Unless a good initial guess of the free surface is set, the adaptive mesh methods are hard to converge if inhomogeneous soils or complicated configurations are present. Moreover, the seepage analysis is usually coupled with the stress analysis, where the different meshes for the seepage analysis and the stress analysis would incur great troubles. In the setting of the classical finite element method, as a result, the adaptive mesh methods appear to have a tendency to give way to the fixed mesh methods.

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The fixed mesh methods, whether the residual flow procedure (RFP) [1] or the variational inequality methods [2,3], blur the dry domain and the wet domain. The free surface is automatically identified once the problem is solved out. Although the RFP formulation is heuristic, 2- and 3-dimensional numerical examples [4] have shown it is very effective. Besides, Westbrook [5] showed that the Alt inequality method and the RFP have much in common when the numerical schemes resulting from these views are examined. Recently, Zheng et al. [6] gave a variational inequality formulation to the RFP [1,7].

Comprehensive reviews of variational inequality methods for free surface seepage problems are given by Oden and Kikuchi [8] and Bruch [9]. By specifying the potential seepage surface as Signorini's boundary condition, Zheng et al. [10] proposed a new variational inequality formulation that eliminates singularity of the seepage point, mitigating the mesh dependence. Later on, Chen et al. [11] generalized the formulation to the non-steady seepage flow.

Rigorous as they are, the variational inequality methods require more mathematical training than the average engineer receives in his formal education. With maturity of the mesh-free methods, as a result, the idea in the adaptive mesh methods is adopted again to solve free boundary value problems including the unconfined seepage problems. We mention that some methods based on the finite element methods seemingly analyze the unconfined seepage problems on fixed meshes, such as [12], but they actually utilize the strategies of the adaptive mesh methods in calculating the contribution of those elements cut by the free surface to the flow matrix.

In addition to the finite element methods, some other numerical methods are also used in the unconfined seepage problems. These methods have their own strengths and weaknesses. For example, the finite volume method by Darbandi and Torabi [13] assures mass conservation over cells but requires that the grid match the free surface. Starting also from the mass conservation equation of integration form, the finite difference method by Bardet and Tobita [14] would encounter troubles in enforcing the boundary conditions if the problem domain is complicated in shape. Highly accurate as it is, the residual velocity method by Zhang and Jiang [15] is suited only for the situations where the free surface does not undergo drastic singularity or changes during iteration. With no need to pay much attention to the location of the free surface in iteration, the level-set method by Herreros et al. [16], has limited precision; and so on.

Now that the mesh fetters have been broken due to the development of the mesh-free methods, which are cut out for the free boundary problems including the unconfined flow problems. As far as we know, Li et al. [17], firstly adopted the Element-Free Galerkin Methods (EFGM) to solve unconfined seepage problems. In the procedure, they selected the moving least squares (MLS) method with singular weight functions, yielding the shape functions with interpolation property. Using the shape functions with interpolation property simply assures the satisfaction of the essential boundary condition at the boundary nodes but not on the whole essential boundary. From the results given in their paper, as a result, the solution precision is very limited and qualified only for homogeneous embankments. Besides, the nodes in the procedure usually have to be added or deleted during iteration. In principle, adding or deleting nodes in the EFGM is feasible. Nevertheless, it is not easy how to add nodes so that accuracy and numerical stability are guaranteed, because the configuration of nodes in the MLS has quite a salient effect on the accuracy of the function to be approximated, see examples given in [18].

To exploit the properties of the MLS, we formulate the variational forms of the unconfined seepage problem in the setting of the numerical manifold method (NMM) [19], where the mathematical cover (MC) is composed of the regularly-deployed nodes and the associated shape function supports. Considering the moving free surface in iteration might not match the regularly-deployed nodes, the proposed variational forms enforce both the essential boundary condition and the material interface continuity condition. So, the mesh adjustment is unnecessary during iteration.

As we know, another crux in the analysis of unconfined seepage flow is the treatment of free surfaces with singularity. For example, when a free surface penetrates a material interface that separates two media with quite different seepage properties, it will have an abrupt change, causing solution hard to converge. Even for a very neat problem as shown in Fig. 12(a) below, the results given by those popular procedures differ considerably from each other, with almost all having very large errors. In order to improve the convergence in treating singular free surfaces, a new strategy for updating the free surface in iteration is put forward. From the typical examples, it will be seen that not only is the strategy able to achieve very high accuracy, but the introduction of the NMM is necessary for treating those singular free surfaces as well. Besides, we also analyzed a practical seepage problem of an earth and rockfill dam, where an internal seepage face develops along the material interface between a sloping core and the rest of the dam. Such problems are frequently encountered in the seepage analysis of earth and rockfill dams, but seldom touched in the literature.

2. Numerical manifold space

We first recapitulate the numerical manifold method (NMM) invented by Shi [19]. Suppose Ω is the problem domain. To be clear, we confine ourselves to two dimensional cases. Ω may keep invariant as in most applications, may deform in space due to being loaded, or may have some internal or external free boundaries that are unknown *a priori*. Internal free boundaries arise in the elastic–plastic analysis, the unsaturated seepage analysis, and so on.

To fit itself to the uncertainty of Ω, the NMM introduces two cover systems, what are referred to as the mathematical cover (MC) and the physical cover (PC), respectively. The MC is a collection of simply connected domains $\{M_i\}$, $i = 1, \ldots, m$, with each M_i called a mathematical patch, and m the number of all mathematical patches. $\{M_i\}$ must be large enough in size or sufficient in number to cover the whole Ω. Using the components of Ω, including the boundary, the material