Applied Mathematical Modelling 39 (2015) 881–893

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Practical application of empirical formulation of the stress concentration factor around equally sized dual spherical cavities to aluminum die cast

Sujit Bidhar [a,*], Osamu Kuwazuru [b], Yoshinori Shiihara [c], Yoshihiko Hangai [d], Takao Utsunomiya [e], Ikumu Watanabe [a], Nobuhiro Yoshikawa [c]

[a] National Institute for Material Science, Research Center for Strategic Material Unit, Sengen 1-2-1 Tsukuba, Ibaraki 305-0047, Japan
[b] Department of Nuclear Power and Energy Safety, University of Fukui, 3-9-1 Bunkyo, Fukui-shi, Fukui 910-8507, Japan
[c] Institute of Industrial Science, The University of Tokyo, 4-6-1 Komaba, Meguro, Tokyo 153-8505, Japan
[d] Department of Mechanical System Engineering, Gunma University, 1-5-1 Tenjin-cho, Kiryu-shi, Gunma 376-8515, Japan
[e] Research Organization of Advanced Engineering, Shibuara Institute of Technology, 307 Fukasaku, Minuma-ku, Saitama-shi, Saitama 337-8570, Japan

ARTICLE INFO

Article history:
Received 15 January 2013
Received in revised form 4 July 2014
Accepted 10 July 2014
Available online 22 July 2014

Keywords:
Finite element method
Stress concentration
Porosity
Dual cavity
Aluminum die cast

ABSTRACT

An empirical method is developed for obtaining the stress concentration factor for a pair of equally sized spherical cavities embedded in a large continuum in three-dimensional space. For practical applications such as die-cast materials containing many pores, we construct a simple and robust closed-form equation to evaluate the stress concentration factor considering the interaction between two cavities. The stress concentration factor can be used to evaluate the effect of pores on the material strength and the probable location of pores that will initiate a fatigue crack. Three-dimensional finite element linear elastic analysis was carried out to evaluate the stress concentration factors for arbitrary locations of the two cavities. The effects of the inter-cavity distance and the orientation of the inter-cavity axis with respect to the loading direction on the stress concentration factor are numerically obtained by systematically changing each of these parameters. Two empirical equations are proposed to fit the stress concentration factor data calculated by finite element analysis after considering various boundary conditions from a mechanical standpoint, and the parameters of the empirical formula are obtained by non-linear curve fitting with regression analysis.

1. Introduction

The stress enhancement caused by inter-cavity interaction is very important from a fatigue strength point of view, since it facilitates fatigue crack initiation [1,2] and degrades the long-term reliability of metal members under dynamic loading. Cavities of irregular shape and size are usually formed during manufacturing, but evaluating the stress field around an irregular boundary is difficult, and closed-form solutions are not available for such geometries. For simplicity, such voids

* Corresponding author.
E-mail addresses: sujit@telu.iis.u-tokyo.ac.jp (S. Bidhar), kuwa@u-fukui.ac.jp (O. Kuwazuru), nori@telu.iis.u-tokyo.ac.jp (Y. Shiihara), hanhan@gunma-u.ac.jp (Y. Hangai), utunomiy@sic.shibaura-it.ac.jp (T. Utsunomiya), WATANABE.Ikumu@nims.go.jp (I. Watanabe), yoshi@telu.iis.u-tokyo.ac.jp (N. Yoshikawa).

http://dx.doi.org/10.1016/j.apm.2014.07.005
0307-904X/

are assumed to be regular spherical cavities. Therefore, it is important to study the interference between stress fields caused by multiple spherical cavities. If these inter-cavity interactions can be modeled using simple mathematical functions, the models may find practical application in structural materials such as pressure die cast aluminum alloys in which micro-porosity is inherent to the casting process. Aluminum die cast materials are widely used in the automotive industry. The fatigue performance of these components is greatly influenced by the presence of cavities [3–7]. Therefore, modeling the three-dimensional stress field around a spherical cavity pair would help identify the stress concentrated regions in actual die cast materials containing many cavities in the form of gas pores and shrinkage pores.

An aluminum die cast part contains a large number of gas pores of irregular shape and size. It would be extremely difficult to consider the interactions among all of the pores at once. A good first step, however, would be to evaluate an empirical function representing the stress concentration factor for a pair of ideal spherical cavities of the same size. Then, this function can be modified to approximate stress concentrations around irregularly shaped cavities. Therefore, in this study, a systematic numerical evaluation of the stress concentration factor for a pair of spherical cavities in three dimensions is carried out. Many researchers have dealt with the problem of a single spherical cavity in an infinite solid by using spheroidal harmonics [8]. Several other numerical methods have been suggested to solve the problem of an infinite solid containing a pair of cavities or inclusions [9–15]. Usually, such methods employ complex stress functions and series of bi-spherical harmonics or multi-pore expansions to describe the stress field around a pair of cavities [9–12]. Stenberg and Sadowsky used Papcovich–Boussinesq displacement functions in spherical dipolar coordinates to solve for the stress distribution around a spherical cavity pair in an infinite solid [9]. Miyamoto et al. [12,13] used spherical coordinates and Papkovich–Neuber stress functions. Such mathematical functions often involve infinite series, which demands more computation time to evaluate the various coefficients. Accuracy is often affected by the number of terms selected in the infinite series. Convergence is not guaranteed when the inter-cavity distance is small [14,15]. Analytical solutions are also unavailable for differently sized cavity pairs and asymmetrical geometries, such as when the inter-cavity axis is not perpendicular to the loading direction. Therefore, in this paper, we attempted to develop an empirical formula for a pair of spherical cavities based on the numerical results of finite element analyses. Parametric studies are then made for a spherical cavity pair by systematically changing both the orientation of the inter-cavity axis with respect to the loading direction and the inter-cavity distance. A similar work is also done to describe stress concentration factor as a function of pore parameters using a simple power function [16]. In this paper we show two types of formula for the function expressing the effect of distance of two pores. One is a simple power function [16], and the other is a comprehensive double exponential function. The difference between two formulae and their accuracy is discussed. We have enhanced the accuracy by using a double exponential function for the effect of distance. The parameters of the empirical formula are optimized through exclusive and coupled search. The practical significance of both empirical formulae, *i.e.*, double exponential function and power function, are discussed in order to provide a qualitative guide line to industry personnel for setting an acceptance limit to die cast parts.

2. Methodology

2.1. Equal size dual cavity problem

Two spherical cavities of equal size are introduced into a large solid cylindrical continuum. The radius and height of the cylindrical volume are forty times the cavity diameter, in order to avoid any free boundary effect. Fig. 1 shows a cross-sectional

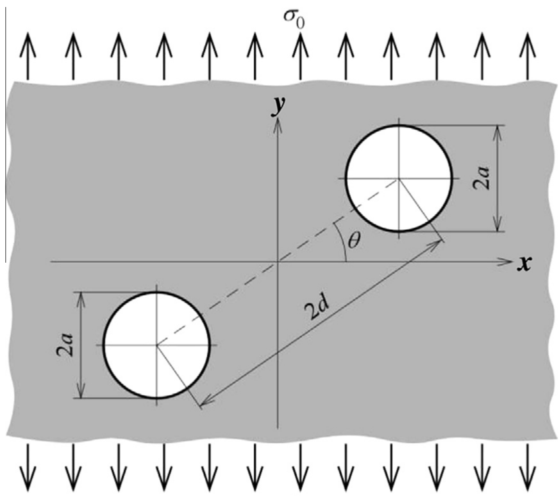

Fig. 1. Layout of a pair of identical spherical cavities in an infinite continuum.