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On the optimal modeling and evaluation of job shops with a total weighted tardiness objective: Constraint programming vs. mixed integer programming

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ABSTRACT

In this study we consider the mapping of the main characteristics, i.e., the structural properties, of a classical job shop problem onto well-known combinatorial techniques, i.e., positional sets, disjunctive graphs, and linear orderings. We procedurally formulate three different models in terms of mixed integer programming (MIP) and constraint programming (CP) paradigms. We utilize the properties of positional sets and disjunctive graphs to construct tight MIP formulations in an efficient manner. In addition, the properties are retrieved by the polyhedral structures of the linear ordering and they are defined on a disjunctive graph to facilitate the formulation of the CP model and to reduce the number of dominant variables. The proposed models are solved and their computational performance levels are compared with well-known benchmarks in the job shop research area using IBM ILog Cplex software. We provide a more explicit analogy of the applicability of the proposed models based on parameters such as time efficiency, thereby producing strong bounds, as well as the expressive power of the modeling process. We also discuss the results to determine the best formulation, which is computationally efficient and structurally parsimonious with respect to different criteria.

1. Introduction

In general, obtaining an optimal solution for a scheduling problem is non-trivial. In this study, we handle the job shop scheduling problem from the viewpoint of a practitioner whose aim is to scrutinize a scheduling problem in a different combinatorial optimization framework. A practitioner with an inappropriate outlook would formulate a scheduling problem as a mixed integer programming (MIP) problem and use the default settings in commercially available software, instead of using problem-specific algorithms. Therefore, we initially focus on obtaining the optimal solution of the job shop scheduling problem with the total weighted tardiness objective (JS-TWT) using mathematically robust procedures such as MIP and constraint programming (CP).

It should be noted that few researchers have revisited the capacity of MIP given the many hardware and software advances that have been made recently, thus we consider these formulations in this context. In particular, the mathematical models used for single machine scheduling problems have received far less attention than heuristics and after nearly 50 years, the composition of the models introduced by Wagner [1], and Manne [2] still retain their original forms but they

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have also entered neglect. This might be due partially to their low efficiency and/or the absence of high-tech processors, but there are few model development techniques based on these compositions. Nevertheless, the complexity of models depends on the number of jobs and machines, but also on the formulation framework.

The classical job shop problem has many practical applications in processes where a number of tasks need to be scheduled in a specific order. These applications range from production and manufacturing environments, e.g., a semiconductor factory, to service-based industries such as hospitals or transportation systems, e.g., railways. A classical job shop problem that is independent of any objective form is known to be an NP-complete problem [3]. Thus, before the development of an effective algorithm, the different *structural properties* of difficult scheduling problems such as JS-TWT should be addressed based only on the nuances of general-purpose models [4], e.g., mathematical models. This can be achieved by traditional combinatorics, but mapping the mathematical structures of these combinatorial techniques onto JS-TWT is a major challenge.

In the widely used three-field notation of Graham, JS-TWT is written as: $J_m \| \sum w_j T_j$ where $T_j = \max(0, c_j - d_j)$. The basic characteristics of a classical job shop are defined as follows. The processing times are known, fixed, and independent of the sequence. All jobs are ready to be processed at time zero. Recirculation is not allowed, i.e., all jobs should meet the machines only once. Preemption is not permitted, i.e., once an operation has started on a machine, it must be completed before another operation may be started on that machine. Only one job may be processed by a machine at any instant in time and each job should encounter all machines.

The first study to describe the formulation of scheduling problems with a mathematical notation was published in 1959. Early studies mostly addressed single machine scheduling as well as multi-machine scheduling. MIP formulations were developed and treated as powerful combinatorics to investigate the computational aspects and structural properties of scheduling problems. The first attempt to minimize the makespan with three machines in series, which is known as the flow shop problem, was reported by Wagner [1]. In this formulation, the key binary variables are x_{jk}; 1 if job j is placed in the kth position in the sequence. In previous studies, these variables are known as assignment variables or positioning variables. Another early study Manne [2] focused on the job shop problem with the aim of minimizing the makespan. In this formulation, the key binary variables are y_{ij}; 1 if job i precedes j in the schedule. These variables are also referred to as precedence variables and they are used in combination with big-M as a large constant to formulate precedence between jobs or operations (disjunctive constraints). In addition, Baker and Keller [5] used these variables to formulate a MIP model for single machine scheduling with the tardiness objective.

The number of variables and solution spaces are essential features of mathematical or MIP models. Thus, increasing the dimensions of variables or using an ill-defined solution space in the techniques designed to handle MIPs may lead to their collapse and the failure to obtain the optimum as an exponential function of time. In the last three decades, in order to alleviate this problem, computer scientists have developed a combinatorial framework with different variable data structures, graph theory, artificial inference, novel computer programming techniques, and branch and bound trees. This framework is called constraint programming (CP) and it is now used extensively for solving complex scheduling problems. In general, the main features of CP are: (1) the ability to handle heterogeneous constraints, multiple disjunctions, and non-convex solution spaces; (2) it is independent of the problem domain size; and (3) it allows the use of an optimization programming language (OPL) [6]. In general, CP solvers are designed according to two concepts: propagation and filtering (or consistency and backtracking). In the first step, the solver must logically eliminate some of the values in the feasibility set whereas those that are compatible with the model are retained. In the second step, the solver must then challenge its previously selected values and test other values [7]. However, existing CP solvers cannot provide a global view of the problem and they cannot exploit the lower bounds provided by linear relaxations, while in some cases, the artificial intelligence used in CP solvers fails to determine the optimal node within a reasonable amount of time. Thus, there is a trade-off between the careful selection of MIP and CP models during the formulation of different aspects of the problem.

In contrast to job shops with the makespan objective, the solution procedures for the JS-TWT are very limited. We consider that it is convenient to divide previous JS-TWT research into three categories: exact methods, heuristics, and theoretical methods. For example, the branch and bound algorithm developed by Singer and Pinedo [8] belongs to the first category. Heuristics are also divided into the local search approach [9,10] and the shifting bottleneck procedure [11]. Theoretical approaches such as dynamic programming algorithms [12], as well as the first category, may be more appropriate for obtaining deep insights into the structural properties and complexity of the JS-TWT.

The main goal of this study is to map the structural properties of the JS-TWT onto the well-known combinatorics, i.e., positional sets, disjunctive graphs, and linear orderings. The positional sets and the disjunctive graphs inherently possess key properties that precisely define the convex hull of the MIP models. The origin of CP can be traced back to graph theory, thus linear ordering properties are implemented in the present study to formulate a disjunctive graph in an efficient manner and to translate it into an appropriate CP model. The properties are retrieved by the polyhedral structure of the linear ordering and defined on a disjunctive graph, thereby reducing the number of dominant variables in the CP model. In this study, we also test the logic used by the CP solver when the processing times of different jobs sets have few variations and the schedule is tight. A simple study of these models was conducted previously by Namakshenas and Sahraeian [13]. However, we completely revise and tighten the MIP formulations presented in [13] by introducing new binary variables and different constraint sets.

The remainder of this paper is organized as follows. Section 2 procedurally discusses the process of generating MIP formulations based on the aforementioned properties. Section 3 presents the linear ordering properties and the translation of

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