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Existence of nonoscillatory solutions of first-order nonlinear neutral differential equations

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ABSTRACT

In this article, we obtain sufficient conditions for first-order nonlinear neutral differential equations to have nonoscillatory solutions for different ranges of $p_1(t)$ and $p_2(t,\xi)$. We use the Knaster–Tarski fixed point theorem to obtain new sufficient conditions. We give an example to illustrate the applicability of our results.

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1. Introduction

In this work, we consider the following first-order nonlinear neutral differential equations:

$$\left[[x(t) - p_1(t)x(t-\tau)]^{\gamma}\right]' + Q_1(t)G(x(t-\sigma)) = 0, \tag{1}$$

$$\left[[x(t) - p_1(t)x(t-\tau)]^{\gamma}\right]' + \int_c^d Q_2(t,\xi)G(x(t-\xi))d\xi = 0 \tag{2}$$

and

$$\left[\left[x(t) - \int_a^b p_2(t,\xi)x(t-\xi)d\xi\right]^{\gamma}\right]' + \int_c^d Q_2(t,\xi)G(x(t-\xi))d\xi = 0, \tag{3}$$

where γ is a ratio of odd positive integers, $\tau > 0, \sigma \geqslant 0, d > c \geqslant 0, b > a \geqslant 0, p_1 \in C([t_0,\infty),\mathbb{R}), p_2 \in C([t_0,\infty) \times [a,b],[0,\infty)), Q_1 \in C([t_0,\infty),[0,\infty)), Q_2 \in C([t_0,\infty) \times [c,d],[0,\infty)), G \in C(\mathbb{R},\mathbb{R})$, and $xG(x) > 0$ for $x \neq 0$. We give some new sufficient conditions for the existence of nonoscillatory solutions of (1)–(3).

In the literature, there are some papers concerning the existence of nonoscillatory solutions of first-order neutral differential equations. For instance, Zhang et al. [1] considered the first-order linear neutral differential equations of the form

$$\frac{d}{dt}[x(t) + P(t)x(t-\tau)] + Q_1(t)x(t-\sigma_1) - Q_2(t)x(t-\sigma_2) = 0 \tag{4}$$

and they established some sufficient conditions for the existence of nonoscillatory solutions of (4). Later, the existence of nonoscillatory solutions of first-order and second-order neutral differential equations with distributed deviating arguments

$$\frac{d^k}{dt^k}[x(t) + P(t)x(t-\tau)] + \int_a^b q_1(t,\xi)x(t-\xi)d\xi - \int_c^d q_2(t,\mu)x(t-\mu)d\mu = 0, \tag{5}$$

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was investigated by Candan and Dahiya [2]. Since Eqs. (4) and (5) are linear, they used the Banach contraction principle to prove their results. However, since (1)–(3) are nonlinear equations, we cannot use the Banach contraction principle. These points make the present article important for the reader studying this field. For some other related articles, we refer the reader to the papers [3–10] and the references cited therein. For books, we refer the reader to [11–15].

Let $m_1 = \max\{\tau, \sigma\}$. By a solution of (1) we mean a function $x \in C([t_1 - m_1, \infty), \mathbb{R})$ for some $t_1 \geqslant t_0$ such that $[x(t) - p_1(t)x(t-\tau)]^\gamma$ is continuously differentiable on $[t_1, \infty)$ and (1) is satisfied for $t \geqslant t_1$. Similarly, let $m_2 = \max\{\tau, d\}$. By a solution of (2) we mean a function $x \in C([t_1 - m_2, \infty), \mathbb{R})$ for some $t_1 \geqslant t_0$ such that $[x(t) - p_1(t)x(t-\tau)]^\gamma$ is continuously differentiable on $[t_1, \infty)$ and (2) is satisfied for $t \geqslant t_1$. Finally, let $m_3 = \max\{b, d\}$. By a solution of (3) we mean a function $x \in C([t_1 - m_3, \infty), \mathbb{R})$ for some $t_1 \geqslant t_0$ such that $[x(t) - \int_a^b p_2(t, \xi)x(t-\xi)d\xi]^\gamma$ is continuously differentiable on $[t_1, \infty)$ and (3) is satisfied for $t \geqslant t_1$.

As is customary, a solution of (1)–(3) is said to be oscillatory if it is neither eventually positive or negative. Otherwise, it is called nonoscillatory.

The following fixed point theorem will be used in proofs.

Theorem 1 (*Knaster–Tarski Fixed Point Theorem [11]*). *Let X be a partially ordered Banach space with ordering $\leqslant$. Let M be a subset of X with the following properties: the infimum of M belongs to M and every nonempty subset of M has a supremum which belongs to M. Let $T : M \to M$ be an increasing mapping, i.e., $x \leqslant y$ implies $Tx \leqslant Ty$. Then T has a fixed point in M.*

2. The main results

Theorem 2. *Assume that $0 \leqslant p_1(t) \leqslant p < 1$, G is nondecreasing and*

$$\int_{t_0}^{\infty} Q_1(s)ds < \infty. \tag{6}$$

Then (1) has a bounded nonoscillatory solution.

Proof. Let Y be the set of all real-valued bounded continuous functions on $[t_0, \infty)$ with the sup norm. We can define a partial ordering as follows: for given $x_1, x_2 \in Y$, $x_1 \leqslant x_2$ means that $x_1(t) \leqslant x_2(t)$ for $t \geqslant t_0$. Set

$$S = \{x \in Y : C_1 \leqslant x(t) \leqslant C_2,\ t \geqslant t_0\},$$

where C_1 and C_2 are positive constants such that

$$C_1 \leq \alpha < (1-p)C_2.$$

If $\tilde{x}_1(t) = C_1,\ t \geqslant t_0$, then $\tilde{x}_1 \in S$ and $\tilde{x}_1 = \inf S$. In addition, if $\emptyset \subset S^* \subset S$, then

$$S^* = \{x \in Y : \lambda \leqslant x(t) \leqslant \mu,\ C_1 \leqslant \lambda,\ \mu \leqslant C_2,\ t \geqslant t_0\}.$$

Let $\tilde{x}_2(t) = \mu_0 = \sup\{\mu : C_1 \leqslant \mu \leqslant C_2,\ t \geqslant t_0\}$. Then $\tilde{x}_2 \in S$ and $\tilde{x}_2 = \sup S^*$. From condition (6) there exists $t_1 > t_0$ with

$$t_1 \geqslant t_0 + \max\{\tau, \sigma\} \tag{7}$$

sufficiently large that

$$\int_t^{\infty} Q_1(s)ds \leq \frac{[(1-p)C_2]^\gamma - \alpha^\gamma}{G(C_2)}, \quad t \geqslant t_1. \tag{8}$$

For $x \in S$, we define

$$(Tx)(t) = \begin{cases} p_1(t)x(t-\tau) + \left[\alpha^\gamma + \displaystyle\int_t^{\infty} Q_1(s)G(x(s-\sigma))ds\right]^{\frac{1}{\gamma}}, & t \geqslant t_1 \\ (Tx)(t_1), \quad t_0 \leqslant t \leqslant t_1. & \end{cases}$$

Thus Tx is a real-valued continuous function on $[t_0, \infty)$ for every $x \in S$. For $t \geqslant t_1$ and $x \in S$, by making use of (8), we obtain

$$\begin{aligned} (Tx)(t) &\leqslant p\,C_2 + \left[\alpha^\gamma + G(C_2)\int_t^{\infty} Q_1(s)ds\right]^{\frac{1}{\gamma}} \\ &\leqslant p\,C_2 + \left[\alpha^\gamma + G(C_2)\frac{[(1-p)C_2]^\gamma - \alpha^\gamma}{G(C_2)}\right]^{\frac{1}{\gamma}} \\ &\leqslant C_2 \end{aligned}$$

and

$$(Tx)(t) \geqslant \alpha \geqslant C_1.$$

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