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Multi-Projection Method for Volterra Integral Equations at the Collocation Points

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Abstract

This study tries to reach a new order of convergence at the collocation points. For this reason we estimated the solution of volterra integral equation to lower and upper solutions on the $S_{m-1}^{(-1)}$ whose elements are spline polynomials of degree $m-1$. Since the upper solution is based on the iterated method the superconvergence can not be accrued at the collocation points. In fact, the upper solution at these points is equal to zero. Therefore, the lower solution which is obtained from the linear system of equations is supposed as an approximating solution at the collocation points and the order of convergence at these points for $m = 1$ is 2 and otherwise $m + 2$.

Keywords: Volterra Integral Equation, Collocation Points, Multi-Projection Method.

1. Introduction

Consider the linear Volterra integral equation of the second kind as follows

$$y(x) = \int_0^x K(x,t)y(t)dt + f(x), \quad x \in J = [0,T] \quad (T > 0), \tag{1}$$

where J is a compact interval, $f \in C(J)$ and $K \in C(S)(S = \{(x,t)|0 \le t \le x \le T\})$. Supposing $\mathbf{K}y = \int_0^x K(x,t)y(t)dt$ leads to obtain the operator equation for (1) as $y = \mathbf{K}y + f$. Let $t_n = nh$ such that partition of J is $\sigma_0 = [t_0, t_1]$ and $\sigma_n = (t_n, t_{n+1}]$, $n = 1, \dots, N-1$. According to [1] we suppose that $Z_N = \{t_n | n = 1, \dots, N-1\}$, $\overline{Z}_N = Z_N \cup T$ and the approximating space is as follows.

$$S_{m-1}^{(-1)}(Z_N) = \{u : u|_{\sigma_n} = u_n \in \pi_{m-1}, n = 0, \dots, N-1, m \ge 1\},$$

whose elements are piecewise polynomials of degree $m-1$. We will compute the approximating solution which belong to $S_{m-1}^{(-1)}(Z_N)$. Assume a set of collocation points as $X_n = \{t_n + c_j h | 0 \le c_1 \le \dots \le c_m \le 1\}$ such that the collocation parameters c_j are the Gauss points, which are the zeros of the shifted Legendre polynomial $P_m(2x-1)$. Projection methods have an important role to solve integral equations. Collocation, the most important projection method, has been considered in the recent decades. This method which has been used to solve the first kind of Volterra integral equation [2] and the second kind [1] leads to efficient solutions. In [1] it has been shown that simple collocation method can not have the property of local superconvergence of (optimal) order $2m$ at the collocation points (see Theorem 2.1 in [1]). In fact, Brunner and Yan (1996) [3] have shown that by the use of the iterated collocation the order of convergence at the collocation points is $m+1$. Hence, in this study we use a linear system of equations, which was obtained from an idea of multi-projection method. This linear system of equations leads to obtain an approximating solution of (1) which has the order of convergence for $m = 1$ and $m \ge 2$ as 2 and $m+2$ at the collocation points respectively. The multi-projection method [4] for solving integral equations leads to decompose the kernel of integral equation to the lower and upper resolutions. According to these decompositions the final solution is decomposed to lower and upper resolutions that we can see in the matrix representation form [5]. The lower solution is obtained from a linear system of equations and the upper solution is based on the lower solution.

2. Multi-Projection Method

In this section we represent how to divide the main solution to the lower and upper solutions which leads to the new order of convergence for the elements X_n. Let $\mathcal{P}_n y = u$ be an interpolation projection such that converges to the identity operator which denoted by I. The interpolation projection has some properties as $\mathcal{P}_n\mathcal{P}_n = \mathcal{P}_n$, $\mathcal{P}_n(I - \mathcal{P}_n) = 0$ and $(I - \mathcal{P}_n)(I - \mathcal{P}_n) = (I - \mathcal{P}_n)$ so that they are used in this section. We replace $\mathbf{K}$ with finite rank operator and after that the approximating solution for (1) was calculated by solving a system of linear equations. Hence, in collocation method $\mathbf{K}$ was replaced with $\mathbf{K}_n^c = \mathcal{P}_n\mathbf{K}\mathcal{P}_n$

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