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Firm behavior under illiquidity risk

Eric A. DeVuyst [a,*], Justin Garosi [b], Jiahong Wu [c]

[a] *Agricultural Economics, Oklahoma State University, United States*
[b] *Legislative Analyst Office, State of California, United States*
[c] *Mathematics, Oklahoma State University, United States*

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ABSTRACT

We develop a model of firm behavior in the presence of risk, resource constraints, and a cash flow constraint. Given imperfect capital markets, the producer confronts an uncertain cash flow. Utilizing chance constrained programming, we show that an increase in aversion to liquidity risk can cause an increased allocation to high-risk production alternatives. With a binding cash flow constraint, risk-averse firms appear to demonstrate risk-seeking behavior over losses and risk-averse behavior over gains.

1. Introduction

Various authors have observed that individuals' or firms' tolerance for risk appears to vary depending on the situation. Friedman and Savage (FS) [1] argue that utility functions take an "S-shape", implying risk-aversion over some ranges of wealth and risk-seeking over others. In response to this observation, Kahneman and Tversky (KT) [2] proposed Prospect Theory. Under this alternative normative model, decision makers value gains and losses as opposed to wealth, and decision weights are used instead of probabilities. In contrast to both FS and KT, we demonstrate that apparent risk-seeking behavior can result from constraints on the decision makers' actions and can be consistent with risk-aversion. In our model, we are concerned with the behavior of firms in jeopardy of failing to meet cash flow obligations. We show that the chance of cash flow failure leads to downward sloping expected profits in resource endowments and increasing investment in risky investments as risk increases, i.e., apparent risk-seeking behavior. As Golbe [3] noted, "firms near bankruptcy may take excessive risks", implying at least the possibility of risk-seeking behavior. Our analytical model of risk-averse producer behavior under illiquidity risk demonstrates the rationality of that behavior.

Hakansson [4] also considered a cash flow constraint. His intertemporal model assumed that the firm must cash flow each time period, precluding the possibility of bankruptcy. The effect of Hakansson's borrowing constraints leads to an apparent S-shaped utility function. Mahul [5], assuming a discontinuous payoff matrix, found that risk-neutral firms can appear risk-seeking. Lybbert and Barrett [6] demonstrate that subsistencethresholds lead to an S-shaped reward function and apparent risk-seeking behavior. Some empirical studies are also available. Audia and Greve [7] investigated shipbuilding firms in Japan. They report that risk-seeking was observed in small, limited-resource firms that were facing poor performance.

Our model assumes that producers are risk-averse and that capital markets are incomplete. Greenwald and Stiglitz [8] suggest several justifications for imperfect capital markets and demonstrate that firm behavior is altered as a result. Under our assumptions, the firm, facing imperfect capital markets, faces liquidation if cash flow generation is insufficient to meet

* Corresponding address: 530 Agricultural Hall, Stillwater, OK 74078, United States. Tel.: +1 405 744 6166; fax: +1 405 744 9835.
E-mail address: Eric.devuyst@okstate.edu (E.A. DeVuyst).

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needs—including debt repayment. In the presence of binding cash flow constraints, we derive three primary results: (1) the more averse a producer is to illiquidity, the more risky his/her production strategy; (2) an increase in the riskiness of an output will induce more production of the output; and (3) a decrease in resource endowments can lead to an increase in expected profits.

2. The model

Assume an owner–operator has expected utility given as $EU[W_0(\bar{A}) + \tilde{\pi}]$ where W_0 is initial wealth as a function of a resource endowment $\bar{A}$ and risky net income $\tilde{\pi}$. The owner must allocate the fixed input $\bar{A}$ between a riskless output L and a risky output H, i.e., $\bar{A} = A_L + A_H$ where A_L and A_H are infinitely divisible and $A_L \geq 0$; $A_H \geq 0$. Returns to fixed cost from each unit of A_L are given as $\pi_L > 0$ and are certain. Returns to fixed cost from each unit of A_H are given as $\tilde{\pi}_H$ and are random with mean μ_H. Further, it is assumed that the lower bound of the distribution of $\hat{\pi}_H$ is less than π_L. (Otherwise, L is stochastically dominated and the allocation problem is trivial.) Then, $\tilde{\pi} = A_L\pi_L + A_H\tilde{\pi}_H - f$ where f are fixed costs and are non-cash, such as depreciation. The firm's output is defined implicitly as $F(L, H, x, \bar{A}) = 0$ where x, a vector of variable inputs, cannot substitute for A in the production of L and H.

Given imperfect capital markets, the firm must cash flow or face liquidation. However, cash flow from operations is risky because it is a function of risky profits,[1] i.e., $\widetilde{CF} = A_L\pi_L + A_H\tilde{\pi}_H$. The business must provide sufficient cash flow to cover non-expense cash flows (w), such as principal payments on debt and owner withdrawals or dividends. Mathematically, $\widetilde{CF}(A_L, A_H) - w \geq 0$.[2] Failure to cash flow results in business liquidation. (While some businesses may be able to temporarily forgo production expenses in order to meet cash flow obligations, this option can only be exercised a limited number of periods before profits and cash flows are irreparably damaged.)

Before continuing, the difference between w and f warrants some discussion. Principal payments (w) are not expenses, so are not included in the profit computation. They are, however, demands on the cash of the business. Depreciation (f) is a non-cash expense, so it is included in the profit calculation and not in the cash flow calculation.

If written as a mathematical constraint, a cash flow constraint is problematic because the right-hand side of the constraint is a random variable. Charnes and Cooper [10] developed chance constrained programming to address constraint risk. Using this formulation, positive net cash flow is obtained with some probability β, where $0 \leq \beta \leq 1$.

Mathematically, the management problem is expressed as

$$\begin{aligned} &\max_{A_L, A_H} EU[W_0(\bar{A}) + \tilde{\pi}(A_L, A_H)] \\ &\text{s.t. } A_L + A_H \leq \bar{A} \\ &\text{Prob}\left(\widetilde{CF}(A_L, A_H) - w \geq 0\right) \geq \beta. \end{aligned} \tag{1}$$

In this formulation, the decision maker is willing to accept violation of the cash flow constraint with probability $1 - \beta$ where $0 \leq \beta \leq 1$. The interpretation of $(1 - \beta)$ is as a measure of the decision maker's tolerance for illiquidity. Hakansson [4] assumed that $\beta = 1$, implicitly assuming that an allocation can be found that always cash flows, precluding the possibility of bankruptcy.

The resource constraint in (1) is always binding since the riskless activity dominates idling some of the endowment. Then via substitution, (1) can be rewritten as

$$\begin{aligned} &\max_{A_H} EU[W_0(\bar{A}) + \tilde{\pi}(A_H, \bar{A})] \\ &\text{s.t.} \quad \text{Prob}\left(\widetilde{CF}(A_H, \bar{A}) - w \geq 0\right) \geq \beta. \end{aligned} \tag{2}$$

The constraint in (2) requires that probability that cash flow net of w is non-negative equals or exceeds the decision maker's minimally acceptable probability of cash flow (β). Denote the cumulative density function of cash flow net of w, i.e., $\text{Prob}(\widetilde{CF}(A_H, \bar{A}) - w \leq 0)$, as $cdf_{CF}(0)$. Then, the constraint in (2) can then be rewritten as $1 - cdf_{CF}(0) \geq \beta$.

Setting up the Lagrangian and differentiating with respect to A_H and the Lagrange multiplier λ gives

$$\begin{aligned} &\max_{A_H} \boldsymbol{\mathcal{L}} = EU[W_0(\bar{A}) + \tilde{\pi}(A_H, \bar{A})] - \lambda(1 - cdf_{CF}(0) - \beta) \\ &\frac{\partial \boldsymbol{\mathcal{L}}}{\partial A_H} = \frac{\partial EU[\cdot]}{\partial A_H} - \lambda \frac{\partial cdf_{CF}(0)}{\partial A_H} \leq 0, \quad \frac{\partial \boldsymbol{\mathcal{L}}}{\partial A_H}\lambda = 0 \\ &\frac{\partial \boldsymbol{\mathcal{L}}}{\partial \lambda} = (1 - cdf_{CF}(0) - \beta) \geq 0, \quad \frac{\partial \boldsymbol{\mathcal{L}}}{\partial \lambda} = 0. \end{aligned} \tag{3}$$

[1] With borrowing, cash flow would also be a function of initial wealth W_0. Without loss of generality, we assume that no additional borrowing is possible. The impact of allowing borrowing up to some percentage of initial wealth is a shift upward in cash flow, which has no impact on our qualitative results.

[2] As an anonymous reviewer correctly pointed out, the cash flow constraint represents a short-run shut down rule for the competitive firm in a risky environment. However, unlike those of Sandmo [9] and others, our rule is an assumption of market place conditions rather than derived from a behavioral model. The chance constraint represents a barrier to entry and is also an assumed condition.