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Multiplicity results for second-order two-point boundary value problems with nonlinearities across several eigenvalues

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Abstract

We consider the boundary value problems for nonlinear second-order differential equations of the form

$$u'' + a(t) f(u) = 0, \qquad 0 < t < 1, \qquad u(0) = u(1) = 0.$$

We give conditions on the ratio $f(s)/s$ at infinity and zero that guarantee the existence of solutions with prescribed nodal properties. Then we establish existence and multiplicity results for nodal solutions to the problem. The proofs of our main results are based upon bifurcation techniques.
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1. Introduction

Let

(H0) $a \in C^1[0, 1]$, $a > 0$ for $0 \le t \le 1$.

Let λ_k be the kth eigenvalue of

$$\begin{aligned} &\varphi'' + \lambda a(t)\varphi = 0, \qquad 0 < t < 1 \\ &\varphi(0) = \varphi(1) = 0 \end{aligned} \tag{1.1}$$

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and let φ_k be an eigenfunction corresponding to λ_k. It is well-known that

$$0 < \lambda_1 < \lambda_2 < \cdots < \lambda_k < \lambda_{k+1} < \cdots, \qquad \lim_{k\to\infty} \lambda_k = \infty$$

and that φ_k has exactly $k-1$ zeros in $(0,1)$ (see, e.g., [1, Chapter VI, Section 27]). Very recently, Naito and Tanaka [2] considered the nonlinear second-order boundary value problem

$$\begin{aligned} &u''(t) + a(t) f(u) = 0, \qquad t \in (0,1) \\ &u(0) = u(1) = 0 \end{aligned} \tag{1.2}$$

under the assumptions:
(H1) $f \in C(R)$, $f(s) > 0$ for $s > 0$, $f(-s) = -f(s)$ for $s > 0$, and f is locally Lipschitz continuous on $(0,\infty)$;
(H2) There exist f_0 and f_∞ with $0 \le f_0, f_\infty \le \infty$, and

$$f_0 = \lim_{|s|\to 0} \frac{f(s)}{s}, \qquad f_\infty = \lim_{|s|\to\infty} \frac{f(s)}{s}.$$

Shooting with initial values and using Sturm's comparison theorem, they established the following results.

Theorem A *([2, Theorem 2]). Assume that either $f_0 < \lambda_k < f_\infty$ or $f_\infty < \lambda_k < f_0$ for some $k \in \mathbb{N}$. Then problem (1.2) has a solution u_k which has exactly $k-1$ zeros in $(0,1)$.*

Theorem B *([2, Theorem 3]). Assume that either (i) or (ii) holds for some $k \in \mathbb{N}$:*

(i) $f_0 < \lambda_k < \lambda_{k+1} < f_\infty$;
(ii) $f_\infty < \lambda_k < \lambda_{k+1} < f_0$.

Then problem (1.2) has two solutions u_k, u_{k+1} such that u_k and u_{k+1} have exactly $k-1$ and k zeros, respectively.

In this paper we consider the problem (1.2) under the assumption (H2) and the more general conditions:
(C0) $a \in C[0,1]$, $a > 0$ for $0 \le t \le 1$;
(C1) $f \in C(R)$ with $sf(s) > 0$ for $s \ne 0$.

From (C1) we see that $f(0) = 0$. Moreover, if $0 < f_0 < \infty$, then f is asymptotically linear at 0. We give conditions on the ratio $f(s)/s$ at infinity and zero that guarantee the existence of solutions. In particular, we will show that problem (1.2) has at least $2k$ solutions if the ratio $f(s)/s$ crosses k eigenvalues.

The main results of this paper are the following.

Theorem 1. *Let (C0), (C1) and (H2) hold, and let $f_0, f_\infty \in (0,\infty)$. Assume that either $f_0 < \lambda_k < f_\infty$ or $f_\infty < \lambda_k < f_0$ for some $k \in \mathbb{N}$. Then problem (1.2) has two solutions u_k^+ and u_k^-, u_k^+ has exactly $k-1$ zeros in $(0,1)$ and is positive near $t=0$, and u_k^- has exactly $k-1$ zeros in $(0,1)$ and is negative near $t=0$.*

Theorem 2. *Let (C0), (C1) and (H2) hold, and let $f_0, f_\infty \in (0,\infty)$. Assume that either (i) or (ii) holds for some $k \in \mathbb{N}$ and $j \in \{0\} \cup \mathbb{N}$:*

(i) $f_0 < \lambda_k < \cdots < \lambda_{k+j} < f_\infty$;
(ii) $f_\infty < \lambda_k < \cdots < \lambda_{k+j} < f_0$.

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