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Some remarks on nonlinear consolidation models[☆]

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Abstract

This paper deals with the modelling of nonlinear consolidation phenomena in a homogenous clay changing from an over to a normal consolidation regime. Specifically this paper develops a technical analysis of the model under the assumption of small deformations to derive a new class of models and to show how classical models known in the literature can be regarded as a particular case of the one dealt with in this paper.

Keywords: Consolidation; Nonlinearity; Soil mechanics; Porous media

1. Introduction

Mathematical models of consolidation phenomena have been developed starting from the classical consolidation theory by Terzaghi [1]. This theory is based on the assumption that consolidation of homogeneous clay layers occurs with coefficients of permeability and compressibility constant during the process. The various developments available in the literature, are documented in the review papers [2,3].

Properties of the consolidating clay may vary in space, due to both variations of soil type and stress history. As a consequence experimental results generally show nonhomogeneous and nonlinear behaviors which are not described by simple models. In particular some relevant phenomena can be described only if related to processes moving from an overconsolidated regime to a normally

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consolidated one (i.e. in terms of plasticity theory, from an elastic domain to an elasto-plastic surface). A model to take into account the above phenomenon has been proposed in [4], where this type of behavior has been called a consolidation which changes type. Comparisons with experimental data have validated the above model which shows that change of type substantially modifies the pore pressure profiles. Various applications [5] and simulations obtained by application of the generalized collocation method [6] have shown that substantial quantitative differences can be obtained with respect to the description delivered by relatively simpler models.

The above consideration motivates the contents of this paper which deals with an analysis of the model proposed in [5] to show how such a model generates, under suitable simplifications (mathematical and physical), some relatively simpler models available in the literature. The consolidation model is summarized in Section 2, while Section 3 deals with the above mentioned analysis.

2. Mathematical models for variable soil properties

This section provides, in view of the analysis which will be developed in the next section, a concise description of the mathematical model proposed in [4]. The reader interested in additional information including comparisons with experimental data and simulations for various drainage conditions is referred to the already cited papers [4,5].

The physical–mechanical assumptions which generate the model are the following:

Hypothesis 2.1. The state of the soil is described by the pore pressure and the void ratio. The independent variables are time and vertical coordinate. Therefore, analysis is developed in the one-dimensional case, i.e., the soil is laterally confined and the drainage can only occur in the vertical direction. The dependent variable is the pore pressure.

Hypothesis 2.2. The soil is fully saturated and both phases, water and soil particles, behave as an incompressible medium. The self weight of the porous medium is ignored. Then, during the consolidation process the excess pore pressure can be evaluated by the difference between the effective normal stress at the end of the consolidation σ'_f and its current value σ'. In addition the total vertical stress σ_c generated by the external load is constant in time, i.e. $\sigma_c = \sigma'_f - \sigma'_0$, where σ'_0 is the initial value of σ'.

Hypothesis 2.3. Darcy's law is applicable to the movement of the water through the soil. The constitutive laws linking the void ratio e to the effective stress σ' and the permeability coefficient k are the following:

$$e = e_0 - I_c \log_{10}\left(\frac{\sigma'}{\sigma'_0}\right) = e_0 + c_k \log_{10}\left(\frac{k}{k_0}\right), \tag{2.1}$$

where I_c is the compressibility index, c_k is the permeability index, and e_0 and k_0 correspond to the initial value $\sigma' = \sigma'_0$.

The model is related to suitable dimensionless independent and dependent variables, as well as three dimensionless parameters. In detail,

- t is the time referred to $T = H^2/c_{v_0}$, where H is the drainage path and c_{v_0} the initial consolidation coefficient;
- x is the vertical coordinate referred to H, with $x \in [0, 1]$;