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# On the Bézier variant of generalized Kantorovich type Balazs operators

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**Abstract**

In the present work we define the Bézier variant of the generalized Balazs–Kantorovich operators. The special cases of our operators reduce to some well known operators. We establish the rate of convergence for functions of bounded variation for the generalized operators.


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*Keywords:* Balazs operators; Rate of convergence; Bounded variation; Total variation; Bézier basis

## 1. Introduction

For a real valued function $f$ defined on the interval $[0, \infty)$, Balazs [1] introduced the Bernstein type rational functions, which are defined by

$$R_n(f, x) = \frac{1}{(1 + a_n x)^n} \sum_{k=0}^{n} \binom{n}{k} (a_n x)^k f\left(\frac{k}{b_n}\right), \tag{1}$$

where $a_n$ and $b_n$ are suitably chosen positive numbers independent of $x$.

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The weighted estimates and uniform convergence for the case $a_n = n^{\beta-1}, b_n = n^{\beta}, 0 < \beta \le 2/3$ were investigated in [2]. Recently Ispir and Atakut [3] introduced the generalization of the Balazs operators, which are defined by

$$L_n(f,x) = \frac{1}{\phi_n(a_n x)} \sum_{k=0}^{\infty} \frac{\phi_n^{(k)}(0)}{k!}(a_n x)^k f\left(\frac{k}{b_n}\right), \qquad n \in N, x \ge 0, \tag{2}$$

where $a_n$ and $b_n$ are suitably chosen positive numbers independent of $x$ and $\{\phi_n\}$ is a sequence of functions $\phi_n : C \to C$ satisfying the following conditions:

(i) $\phi_n (n = 1, 2, \ldots)$ is analytic in a domain $D$ containing the disk $B = \{z \in C : |z - b| \le b\}$;
(ii) $\phi_n(0) = 1 (n = 1, 2, \ldots)$;
(iii) for any $x \ge 0$, $\phi_n(x) > 0$ and $\phi_n^{(k)}(0) \ge 0$ for any $n = 1, 2, \ldots$ and $k = 1, 2, \ldots$;
(iv) for every $n = 1, 2, \ldots$,

$$\frac{\phi_n^{(v)}(a_n x)}{n^v \phi_n(a_n x)} = 1 + O\left(\frac{1}{n a_n}\right), \qquad v = 1, 2, 3, 4$$

where $a_n \to 0$, $n a_n \to \infty$ as $n \to \infty$.

In [3] the authors have estimated the order of approximation for the operators defined by (2) and proved a Voronovskaja type asymptotic formula and pointwise convergence in a simultaneous approximation.

The operators defined by (2) are summation type operators, which are not capable of approximating integrable functions. To approximate integrable functions on the interval $[0, \infty)$, we now define the Kantorovich variant of the generalized Balazs type operators as

$$L_n^*(f,x) = n a_n \sum_{k=0}^{\infty} p_{n,k}(x) \int_{I_{n,k}} f(t)\mathrm{d}t, \qquad n \in N, x \ge 0, \tag{3}$$

where $I_{n,k} = [k/na_n, (k+1)/na_n]$, $p_{n,k}(x) = \frac{\phi_n^{(k)}(0)}{k!} \frac{(a_n x)^k}{\phi_n(a_n x)}$ and $x \ge 0$.

**Remark 1.** Some particular cases of the operators are defined as follows:

**Case 1.** If $a_n = 1$ and $\phi_n(x) = \mathrm{e}^{nx}$, then we obtain the Szász–Kantorovich operators, which are defined by

$$S_n^*(f,x) = n\mathrm{e}^{-nx} \sum_{k=0}^{\infty} \frac{(nx)^k}{k!} \int_{k/n}^{(k+1)/n} f(t)\mathrm{d}t, \qquad x \in [0, \infty).$$

**Case 2.** If $\phi_n(x) = (1 + x)^n$, then we obtain the Bernstein Balazs–Kantorovich operators, which are defined by

$$K_n^*(f,x) = n a_n \sum_{k=0}^{n} \binom{n}{k} (a_n x)^k (1 + a_n x)^{-n} \int_{k/na_n}^{(k+1)/na_n} f(t)\mathrm{d}t, \qquad x \in [0, \infty).$$

In computer aided geometric design, Bézier basis functions play an important role. This, along with the recent work on some Bézier variants of well known operators (see [4,5]), motivated us to study further