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Lognormals for SETI, Evolution and Mass Extinctions

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A B S T R A C T

In a series of recent papers (Refs. [1–5,7,8]) and in a book (Ref. [6]), this author suggested a new mathematical theory capable of merging Darwinian Evolution and SETI into a unified statistical framework. In this new vision, Darwinian Evolution, as it unfolded on Earth over the last 3.5 billion years, is defined as just one particular realization of a certain lognormal stochastic process in the number of living species on Earth, whose mean value increased in time exponentially. SETI also may be brought into this vision since the number of communicating civilizations in the Galaxy is given by a lognormal distribution (Statistical Drake Equation).

Now, in this paper we further elaborate on all that particularly with regard to two important topics:

1) The introduction of the general lognormal stochastic process $L(t)$ whose mean value may be an ***arbitrary*** continuous function of the time, $m(t)$, rather than just the exponential $m_{\text{GBM}}(t) = N_0\, e^{\mu t}$ typical of the Geometric Brownian Motion (GBM). This is a considerable generalization of the GBM-based theory used in Refs. [1–8].
2) The particular application of the general stochastic process $L(t)$ to the understanding of Mass Extinctions like the K-Pg one that marked the dinosaurs' end 65 million years ago. We first model this Mass Extinction as a decreasing Geometric Brownian Motion (GBM) extending from the asteroid's impact time all through the ensuing “nuclear winter”. However, this model has a flaw: the “final value” of the GBM cannot have a horizontal tangent, as requested to enable the recovery of life again after this “final extinction value”.
3) That flaw, however, is removed if the rapidly decreasing mean value function of $L(t)$ is the left branch of a parabola extending from the asteroid's impact time all through the ensuing “nuclear winter” and up to the time when the number of living species on Earth started growing up again, as we show mathematically in Section 3.

In conclusion, we have uncovered an important generalization of the GBM into the general lognormal stochastic process $L(t)$, paving the way to a better, future understanding the evolution of life on Exoplanets on the basis of what Evolution unfolded on Earth in the last 3.5 billion years. That will be the goal of further research papers in the future.

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1. Introduction: new statistical mechanisms

This paper describes a new statistical theory casting Evolution and SETI into mathematical terms, rather than just by using words only. The basic statistical tools used in this paper are:

1) The general stochastic process called Lognormal process $L(t)$, embodies an arbitrary function of the time, $M_L(t)$ and an arbitrary positive numeric parameter σ. Thus, the probability density function of this Lognormal process $L(t)$ is denoted by

$$L(t)\_pdf(n; M_L(t), \sigma, t) = \frac{e^{-\frac{[\ln(n)-M_L(t)]^2}{2\sigma^2 t}}}{\sqrt{2\pi}\sigma\sqrt{tn}}, \quad (n \ge 0). \tag{1}$$

2) The mean value of the Lognormal process (1) is then given by

$$m_L(t) = \int_0^\infty n \frac{e^{-\frac{[\ln(n)-M_L(t)]^2}{2\sigma^2 t}}}{\sqrt{2\pi}\sigma\sqrt{tn}}\, dn = e^{M_L(t)} e^{\frac{\sigma^2}{2}t}. \tag{2}$$

When solved for $M_L(t)$, the last equation yields:

$$M_L(t) = \ln(m_L(t)) - \frac{\sigma^2}{2}t. \tag{3}$$

Table 1 shows the main statistical properties of the lognormal stochastic process (1); we skip all the proofs, since those proofs would take many pages, and would also deprive the reader of the "mathematical delight" of checking our results by virtue of some symbolic manipulator like Maxima, Maple, Mathematica, and the like.

3) The particular case of (2) when that mean value is given by the generic exponential:

$$m_{\mathrm{GBM}}(t) = N_0 e^{\mu t} \tag{4}$$

is called Geometric Brownian Motion (GBM), and is widely used in financial mathematics, where it represents the "underlying process" of the stock values (Black–Sholes models). This author also widely used the GBM in his previous mathematical models of Evolution and SETI (Refs. [1–3]), since it was assumed that the growth of the number of ET civilizations in the Galaxy, or, alternatively, the number of living species on Earth over the last 3.5 billion years, ***grew exponentially*** (Malthusian growth). However, this author now realizes that this assumption, symbolized by the exponential mean value (4), was too restrictive. The replacement of (4) by (2) in all models for Evolution and SETI is the main achievement of this paper. Notice also that, upon equating the two right-hand-sides of (2) and (4), we find that

$$e^{M_{\mathrm{GBM}}(t)} e^{(\sigma^2/2)t} = N_0 e^{\mu t}. \tag{5}$$

Solving this equation for $M_{\mathrm{GBM}}(t)$ yields:

$$M_{\mathrm{GBM}}(t) = \ln N_0 + \left(\mu - \frac{\sigma^2}{2}\right)t \tag{6}$$

which is just the "mean value at the exponent" of the well-known pdf of the GBM, that is

$$\mathrm{GBM}(t)\_pdf(n; N_0, \mu, \sigma, t) = \frac{e^{-([\ln(n) - (\ln N_0 + (\mu - (\sigma^2/2))t)]^2/2\sigma^2 t)}}{\sqrt{2\pi}\sigma\sqrt{tn}}, \quad (n \ge 0). \tag{7}$$

We conclude this short description of the GBM as the particular exponential ("Malthusian") case of the general lognormal process (1) by warning the reader that the denomination "Geometric Brownian Motion" is a rather misleading one, since it lets the readers think that GBM are Gaussian (or normal) processes, whereas they are lognormal processes instead.

Table 1
Summary of the properties of the lognormal distribution that applies to the stochastic process $L(t)$=lognormally changing number of ET communicating civilizations in the Galaxy, as well as the number of living species on Earth over the last 3.5 billion years. Clearly, these two different $L(t)$ lognormal stochastic processes may have two different time functions for $M_L(t)$ and two different numerical values for σ, but the equations are the same for both processes, i.e. for the number of ET civilizations in the Galaxy and for the number of living species in the past of Earth.

Stochastic process	$L(t) = \begin{cases} 1)\ Number\ \ of\ \ ET\ \ Civilizations\ \ (in\ \ SETI). \\ 2)\ Number\ \ of\ \ Living\ \ Species\ \ (in\ \ Evolution). \end{cases}$
Probability distribution	**Lognormal distribution of all LOGNORMAL stochastic processes, i.e. the lognormal stochastic processes with ARBITRARY MEAN** $m_L(t)$
Probability density function	$L(t)\_pdf(n; M_L(t), \sigma, t) = \frac{1}{\sqrt{2\pi}\sigma\sqrt{tn}} e^{-([\ln(n) - M_L(t)]^2/2\sigma^2 t)}$ for $n \ge 0$
Mean value	$\langle L(t)\rangle \equiv m_{L(t)} = e^{M_L(t)} e^{(\sigma^2/2)t}$
Variance	$\sigma^2_{L(t)} = e^{2M_L(t)} e^{\sigma^2 t}(e^{\sigma^2 t} - 1)$
Standard Deviation	$\sigma_{L(t)} = e^{M_L(t)} e^{(\sigma^2/2)t}\sqrt{e^{\sigma^2 t} - 1}$
Upper Standard Deviation Curve	$m_{L(t)} + \sigma_{L(t)} = e^{M_L(t)} e^{(\sigma^2/2)t}\left[1 + \sqrt{e^{\sigma^2 t} - 1}\right]$
Lower Standard Deviation Curve	$m_{L(t)} - \sigma_{L(t)} = e^{M_L(t)} e^{(\sigma^2/2)t}\left[1 - \sqrt{e^{\sigma^2 t} - 1}\right]$
All the moments, i.e. k-th moment	$\langle L^k(t)\rangle = e^{kM_L(t)} e^{k^2(\sigma^2/2)t}$
Mode (=abscissa of the lognormal peak)	$n_{\mathrm{mode}} \equiv n_{\mathrm{peak}} = e^{M_L(t)} e^{-\sigma^2 t}$
Value of the Mode Peak	$f_{L(t)}(n_{\mathrm{mode}}) = \frac{1}{\sqrt{2\pi}\sigma\sqrt{t}} e^{-M_L(t)} e^{(\sigma^2/2)t}$
Median (=fifty–fifty probability value for $N(t)$)	$\mathrm{median} = m = e^{M_L(t)}$
Skewness	$\frac{K_3}{(K_2)^{3/2}} = (e^{\sigma^2 t} + 2)\sqrt{e^{\sigma^2 t} - 1}$
Kurtosis	$\frac{K_4}{(K_2)^2} = e^{4\sigma^2 t} + 2e^{3\sigma^2 t} + 3e^{2\sigma^2 t} - 6$