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Calculation and fitting of boundaries between elliptic and hyperbolic singularities of pyramid-type control moment gyros

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A R T I C L E I N F O

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A B S T R A C T

There exists a singularity problem in control moment gyros (CMGs). CMG singularities are classified into two types: hyperbolic and elliptic. Several gimbal steering control methods have been presented to avoid CMG singularities. Hyperbolic singularities can be avoided by null motion, but elliptic singularities cannot. The existing steering control methods are rarely designed by explicitly taking the singularity type into account. In order to effectively avoid elliptic singularities by perturbing gimbal angles, it is desirable to calculate and record the boundaries between elliptic and hyperbolic singularities in advance so that the determined boundaries can be utilized for developing model predictive steering control. To this end, the boundaries between elliptic and hyperbolic singularities of CMGs are calculated and represented in the form of fitted curves. Several numerical examples are presented to determine the perturbation gimbal angles for avoiding elliptic singularities without using singular value decomposition.

1. Introduction

An advantage of control moment gyro (CMG) systems is that they can generate large torques compared to reaction wheels (RWs). There are many single gimbal CMG (SGCMG) array configurations, such as skew type, roof type, symmetric type, and twin type. The pyramid-type CMG system, as shown in Fig. 1, is a typical configuration for a SGCMG system. However, SGCMG systems have a singularity problem. This problem has been studied from various points of view. The singular surface of a pyramid-type CMG system was studied in [1].

The singular surfaces of a CMG system are classified into two types: hyperbolic and elliptic singularities. To overcome the singularity problem of pyramid-type SGCMGs, a number of control schemes have been proposed [2–21]. The logic for singularity avoidance can be generally classified into two categories: gimbal angle path planning and real-time feedback control. In the path planning methods, the gimbal angle trajectories are searched in advance so that the CMG systems do not encounter any singularities during attitude maneuvers [4–6]. However, generally speaking, those methods have high computational cost.

On the other hand, real-time steering logic methods do not have high computational cost but require singularity avoidance techniques because encountering the singularities is not predicted in advance. Real-time singularity avoidance methods for the SGCMGs can be classified into three types: singularity-robust (SR) inverse steering laws [7–10], singular direction avoidance (SDA) methods [11], and gradient methods [7,15,16]. The gradient methods use null motion, which can change the gimbal angles without generating torque. Therefore, null motion can be used to avoid CMG singularities. However, null motion is not perfect because it still cannot avoid elliptic singularities.

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Nomenclature

$\boldsymbol{C}$	Jacobian matrix of the CMG system
$\boldsymbol{C}_i$	$\boldsymbol{C}$ with ith column removed
$\boldsymbol{D}_i$	$\det(\boldsymbol{C}_i)$
E, F, G	elements of the first fundamental form
$\boldsymbol{h}$	angular momentum vector of the CMGs
H	angular momentum magnitude of each CMG $(=1$ Nm s$)$
$\boldsymbol{H}_i$	angular momentum vector of the ith CMG
J	objective function
K	Gaussian radius $(=1/\kappa)$
L, M, N	elements of the 2nd fundamental form
$\mathcal{N}$	matrix consisting of null space basis vectors
$\mathcal{M}$	singularity type determinable matrix
$P(u, v)$	fitted curve function
$Q_i(u, v)$	singular gimbal angles (δ_{s2} or δ_{s4}) determined from (u, v)
$\boldsymbol{S}$	singular surface
$\boldsymbol{u}_s$	singular vector
u, v	independent variables for singular surface $(=\delta_{s1}, \delta_{s3})$
β	skew angle $(=\tan^{-1}\sqrt{2} = 54.73^\circ)$
$\boldsymbol{\delta}$	gimbal angle vector $(=(\delta_1, \delta_2, \delta_3, \delta_4)^T)$
δ_i	gimbal angle of the ith CMG, rad
δ_{si}	singular gimbal angle of the ith CMG
κ	Gaussian curvature

Because the singular surfaces of the pyramid-type SGCMG can be obtained in advance, recorded data of CMG singularities can be used to make both real-time singularity avoidance methods and CMG angular momentum path planning more effective. Takada et al. [20] implemented a singularity avoidance method using the surface cost function as a singularity metric calculated from the singular surface data at 3-D mesh points, and they experimentally showed that their proposed method is effective in real-time singularity avoidance. Sato and Takahashi [21] recorded a singular surface in the form of a set of plane surfaces and applied an A* algorithm [22] to the global singularity avoidance path planning of the CMG angular moment.

Although the above two papers, which are based on recorded singularity data, succeeded in singularity avoidance, the amount of singularity data is considerably large, and the recorded data of the singularities are not precise because recording is limited to the mesh/grid points. Therefore, a more precise and efficient recording method is desired. Moreover, if an attitude tracking maneuver is required, it is desired to use null motion because torque error is not generated by null motion. However, as mentioned above, elliptic singularities cannot be avoided by null motion; thus, torque error is necessary. Even in such a case, a small torque error is still desired for precise attitude tracking maneuvers. In order to compromise between the necessary torque error and precise attitude tracking, a more efficient method of avoiding elliptic singularities is required.

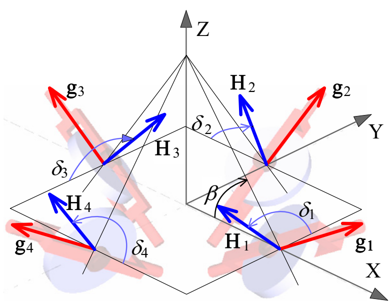

Fig. 1. Pyramid-type CMG.

To overcome the above problems, the boundaries between hyperbolic (passable) and elliptic (impassable) singularities are calculated and recorded in the form of a polynomial function of the gimbal angles using curve fitting techniques. The fitted curves of the boundaries can be used to detect the type of singularity (hyperbolic/elliptic) without singular value decomposition (SVD) and to explicitly determine the gimbal angle perturbation to avoid elliptic singularities during maneuvers. In other words, the boundaries recorded in the form of a function will be able to be used to implement model predictive control (MPC) for singularity avoidance. To the author's best knowledge, no previous papers have dealt with the problem by explicitly determining concrete values of the gimbal angle perturbations to avoid elliptic singularities. Furthermore, the amount of recorded singularity data is discussed and compared with those of Refs. [20] and [21], and the simplicity of the boundaries expressed in the form of polynomial functions is demonstrated by several examples for determining the gimbal angle perturbations to avoid elliptic singularities.

2. Singularities of pyramid-type SGCMGs

In this paper, a pyramid-type SGCMG (as shown in Fig. 1) is considered. In a traditional pyramid-type CMG system, the skew angle β is fixed at $\beta = \tan^{-1}\sqrt{2}$ rad $(=54.73^\circ)$ because for this angle, the momentum envelope, which represents the maximum available angular momentum of the CMG for attitude maneuvers, becomes nearly spherical. The total CMG angular momentum vector for the pyramid mounting of four SGCMGs $\boldsymbol{h}$ is expressed in the spacecraft reference frame as

$$\boldsymbol{h} = \sum_{i=1}^{4} \boldsymbol{H}_i = H\begin{bmatrix} -c\beta \sin \delta_1 \\ \cos \delta_1 \\ s\beta \sin \delta_1 \end{bmatrix} + H\begin{bmatrix} -\cos \delta_2 \\ -c\beta \sin \delta_2 \\ s\beta \sin \delta_2 \end{bmatrix} + H\begin{bmatrix} c\beta \sin \delta_3 \\ -\cos \delta_3 \\ s\beta \sin \delta_3 \end{bmatrix} + H\begin{bmatrix} \cos \delta_4 \\ c\beta \sin \delta_4 \\ s\beta \sin \delta_4 \end{bmatrix} \quad (1)$$