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# Mechanical study on initial profile solution for inflatable reflector made of orthotropic material

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ABSTRACT

The woven fabric membrane materials are widely used in space and terrestrial inflatable reflectors. However the material's anisotropy makes the design analysis more complex. The deviation from the desired shape, so-called "W-profile error", influences the precision of the membrane surface significantly. In this study, a model of an axisymmetric paraboloid surface using membrane theory is established for the purpose of facilitating the surface precision optimization. Analytical solutions for displacements of the reflector are derived. An iteration method of initial reflector profile solution is stated and a finite element (FE) software employed in the solution is presented. A case study is illustrated to make a comparison between numerical and theoretical analyses. Finally, the conclusions are drawn that the analytical method and the FE iterative method for initial profile solution are feasible and efficient.



## 1. Introduction

Ultra-lightweight inflatable reflectors are widely used in space and terrestrial applications [1]. To avoid the material damage introduced by multiple packaging and deployment, the material folding radius should be sufficiently large. The woven fabric membrane materials become appropriate solutions for this application. However, the design analysis becomes more complicated because of the material's anisotropy [2].

The performance of a reflector is mainly determined by the geometrical precision of the membrane surface that a number of researchers have investigated this area. Based on Hencky's study [3], the deviation from parabolic profiles of reflector caused by inflation, namely the so-called "W-profile error" of an inflated membrane reflector [4], has a significant impact to the precision of the membrane surface. The figure error of a reflector fabricated of plane sheet and then deformed to curved surface has been studied by Jenkins and Marker [5] using the von Kármán plate equations. The mechanical analysis of the inflatable arch has been processed with the Sanders' linear thin-shell theory by Plaut et al. [6]. The exact and approximate parabolic shapes of initially curved and flat axisymmetric reflectors as well as their structural characteristics have been discussed by Greschik et al. [7–9]. The analytical approaches with membrane theories [10,11] as well as the Finite Element Method (FEM) [12] have been employed to investigate the deformations and stress distributions of membrane reflectors. The geometric imperfections of a parabolic antenna have been investigated by Naboulsi [13]. To reduce the "W-profile error" and improve the precision of membrane surface, a majority of studies have focused on the active control of the reflector such as optimized gore/seam cable-actuated shape control [4]. However, few studies have been done regarding the precision analyses for the reflectors made of anisotropic fabric membranes. And few studies have been advanced to optimize the initial profile for increasing the precision of the membrane reflector.

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In this paper, for the analyses of inflated reflectors made of an orthotropic material, in Section 2, the axisymmetric parabolic surface model based on membrane theory is established. Then in Section 3, the solutions for displacements of the reflector are derived. An iteration method of initial profile solution for reflector precision improvement is stated in Section 4. A finite element software used in the solution is presented in Section 5. Comparisons between the numerical and theoretical analyses are illustrated through a case study in Section 6. The influences of parameters on the precision of reflectors are investigated and conclusions are drawn at last.

## 2. Fundamental reflector model

To facilitate the surface precision analysis and the optimal design, a thin-shell model of an inflated membrane reflector is presented. The analytical model is a shallow and axisymmetric parabolic cap structure with internal inflation pressures and simply supported boundary conditions. The reflector material is an orthotropic Silver-plated Nylon cloth whose warp fabrics are oriented along the meridian of the cap.

Since the reflector material is very thin, the bending stiffness and the torsional stiffness are negligible. The stiffness of the reflector is mainly from the inflation introduced in-plane strains, here namely the membrane stiffness. Therefore in the analysis process derivation the membrane theory is utilized which ignores the moment and the torque in the membrane plane. The force equilibrium equations are [14]:

$$\begin{cases} \frac{\partial(BN_1)}{\partial\alpha} - N_2\frac{\partial B}{\partial\alpha} + \frac{1}{A}\frac{\partial(A^2N_{12})}{\partial\beta} + ABp_1 = 0 \\ \frac{\partial(AN_2)}{\partial\beta} - N_1\frac{\partial A}{\partial\beta} + \frac{1}{B}\frac{\partial(B^2N_{12})}{\partial\alpha} + ABp_1 = 0 \\ \frac{N_1}{r_1} + \frac{N_2}{r_2} + p_3 = 0 \end{cases} \tag{1}$$

where $\alpha$ and $\beta$ denote the two orthogonal directions in the mid-plane of a membrane element; $A$ and $B$ denote Lame parameters along directions $\alpha$ and $\beta$ respectively; $r_1$ and $r_2$ denote radius of curvature along directions $\alpha$ and $\beta$ respectively; $N_1$ and $N_2$ denote tension forces along directions $\alpha$ and $\beta$ respectively; $N_{12}$ denotes the shear force in the mid-plane; $p_1$, $p_2$ and $p_3$ denote pressures along directions $\alpha$, $\beta$ and normal direction respectively. The relationship between tension forces and stresses are given as

$$N_1 = \sigma_1 t,\ N_2 = \sigma_2 t,\ N_{12} = \sigma_{12} t \tag{2}$$

where $\sigma_1$ and $\sigma_2$ are stresses in the mid-plane of a membrane element along directions $\alpha$ and $\beta$ respectively; $\sigma_{12}$ is the shear stress in the mid-plane; $t$ is the thickness of the membrane element.

The equations of compatibility are [14]

$$\begin{cases} \varepsilon_1 = \frac{1}{A}\frac{\partial u}{\partial\alpha} + \frac{\partial A}{\partial\beta}\frac{v}{AB} + \frac{w}{r_1} \\ \varepsilon_2 = \frac{1}{B}\frac{\partial v}{\partial\beta} + \frac{\partial B}{\partial\alpha}\frac{u}{AB} + \frac{w}{r_2} \\ \varepsilon_{12} = \frac{A}{B}\frac{\partial}{\partial\beta}\left(\frac{u}{A}\right) + \frac{B}{A}\frac{\partial}{\partial\alpha}\left(\frac{v}{B}\right) \end{cases} \tag{3}$$

where $u$, $v$ and $w$ are displacements along directions $\alpha$, $\beta$ and normal; $\varepsilon_1$ and $\varepsilon_2$ are strains along directions $\alpha$ and $\beta$, $\varepsilon_{12}$ is shear strain in the mid-plane.

Fig. 1 shows the definition of the coordinate system and parameters of the reflector model. The piece ABCD in Fig. 1 is an arbitrary element of the membrane. The point P is the intersection of the axis $z$ and the radius of the curve BC while the point O' is the center of the curve AB with the meridian coordinate $\varphi$ and the circumferential coordinate $\theta$ respectively (where $\varphi=0$ at the direction PO' and $\theta=0$ at the direction O'Q). Considering the axisymmetric of displacements and stresses, directions $\alpha$ and $\beta$ can be replaced by the orthogonal curvilinear coordinates $\varphi$ and $\theta$ as the principal coordinates of the element ABCD respectively. The principal radiuses of the element ABCD are shown as $r_1$and $r_2$ in Fig. 1 while the radius of the plane AO'B is given as $r=r_2\sin\varphi$. Therefore, the Lame parameters are namely replaced as $A=r_1$ and $B=r=r_2\sin\varphi$. The inflation pressure $P=p_3$ is the only load taken into consideration.

Eq. (4) can be obtained by substituting aforementioned equations into Eq. (1) as

$$\begin{cases} \frac{\partial(rN_\varphi)}{\partial\varphi} + r_1\frac{\partial N_{\varphi\theta}}{\partial\theta} - N_\theta r_1 \cos\varphi = 0 \\ \frac{\partial(rN_{\varphi\theta})}{\partial\varphi} + r_1\frac{\partial N_\theta}{\partial\theta} + N_{\varphi\theta} r_1 \cos\varphi = 0 \\ \frac{N_\varphi}{r_1} + \frac{N_\theta}{r_2} + P = 0 \end{cases} \tag{4}$$

where $N_\varphi$ and $N_\theta$ are tension forces in coordinates $\varphi$ and $\theta$ respectively shown in Fig. 1; $N_{\varphi\theta}$ is the shear force in the mid-plane shown in Fig. 1.

Considering the axial symmetry, displacement $v$ and variables in the direction $\theta$ can be ignored. From Gauss–Codazzi–Mainardi Equation [14], it is given as

$$\frac{\partial r}{\partial\varphi} = \frac{\partial(r_2 \sin\varphi)}{\partial\varphi} = r_1 \cos\varphi \tag{5}$$

Plug Eq. (5) into Eq. (4) to get

$$\begin{cases} \frac{1}{r_1}\frac{\partial N_\varphi}{\partial\varphi} + \frac{\cot\varphi}{r_2}(N_\varphi - N_\theta) = 0 \\ \frac{N_\varphi}{r_1} + \frac{N_\theta}{r_2} + P = 0 \end{cases} \tag{6}$$

The equilibrium between the inflation pressure and the membrane tension along the axis $z$ is given as

$$P\pi r^2 = N_\varphi 2\pi r \sin\varphi \tag{7}$$

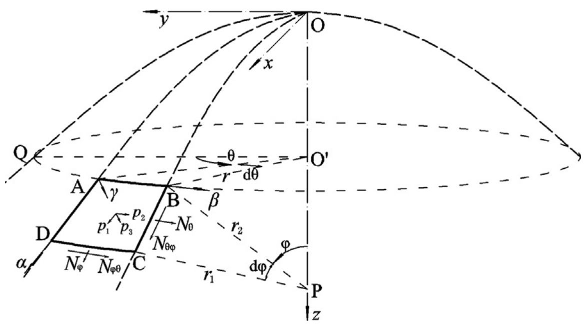


**Fig. 1.** Coordinates and parameters of reflector model.