Acta Astronautica 104 (2014) 324–340

Contents lists available at ScienceDirect

Acta Astronautica

journal homepage: www.elsevier.com/locate/actaastro

Exact analytic solution for the spin-up maneuver of an axially symmetric spacecraft

Jacopo Ventura [a,*], Marcello Romano [b]

[a] Institute of Astronautics, Technische Universität München, 85748 Garching, Germany
[b] Mechanical and Aerospace Engineering Department, Naval Postgraduate School, 700 Dyer Road, Monterey, CA, USA

ARTICLE INFO

Article history:
Received 20 March 2014
Received in revised form
14 July 2014
Accepted 29 July 2014
Available online 13 August 2014

Keywords:
Rigid-body dynamics and kinematics
Spin-up maneuver
Analytic solution

ABSTRACT

The problem of spinning-up an axially symmetric spacecraft subjected to an external torque constant in magnitude and parallel to the symmetry axis is considered. The existing exact analytic solution for an axially symmetric body is applied for the first time to this problem. The proposed solution is valid for any initial conditions of attitude and angular velocity and for any length of time and rotation amplitude. Furthermore, the proposed solution can be numerically evaluated up to any desired level of accuracy. Numerical experiments and comparison with an existing approximated solution and with the integration of the equations of motion are reported in the paper. Finally, a new approximated solution obtained from the exact one is introduced in this paper.

1. Introduction

The problem of spinning-up a spacecraft is of high importance for many space missions. In particular, as many axially symmetric spacecraft are spin-stabilized a spinning-up maneuver is needed in order to bring them from an initial condition typically close to rest to the nominal spinning condition. The spacecraft may need spin-down and spin-up maneuvering capabilities also to perform reorientation maneuvers.

The spin-up problem of spacecraft has been previously studied. For the general case of a spin-up maneuver of a spherically symmetric or an axially symmetric spacecraft subjected to constant torques, several approximated analytic solutions were introduced. Longuski [1] proposes a formulation for angular velocity and Euler angles for both symmetric and near symmetric spacecraft, valid for small angular velocity and rotational angles. Longuski [2] also introduces the approximate analytic solution for the angular momentum vector for near symmetric rigid spacecraft subjected to a constant spinning torque. This solution is given as a function of the angular velocity and Euler angles introduced in Ref. [1] and it is valid for small angular velocity and rotational angles. Another approximated formulation for the angular velocity and Euler angles of a spinning spacecraft, valid for small angles only, is given by Wie [3]. Ayoubi and Longuski [4,5] introduce approximated solutions for the inertial transverse velocity, inertial displacement and axial velocity of a spinning-up axially symmetric spacecraft, under the assumptions of small Euler angles and linear behavior of the spin rate. Longuski [6] considers the case of a spinning spacecraft with constant spinning rate and subjected to transverse body-fixed torques. The author proposes an approximated solution for the attitude, rotational and translational motions of the spacecraft while assuming small angular excursions of the spin axis with respect to an inertially fixed direction. Other authors analyze the motion of a spinning spacecraft in particular situations. Ayoubi and Longuski [7,8] study the asymptotic behavior of the motion of spinning spacecraft subjected to constant torques in all the three axis of the body frame, introducing

---

[*] Corresponding author. Tel.: +49 89 289 16016.
E-mail addresses: jacopo.ventura@tum.de (J. Ventura), mromano@nps.edu (M. Romano).

http://dx.doi.org/10.1016/j.actaastro.2014.07.038
0094-5765/

Nomenclature

A	numeric constant defined in Eq. (17)
a	adimensional parameter defined in Eq. (75)
B	principal body-fixed Cartesian frame, with the third axis directed along the axis of symmetry of the spacecraft
b	adimensional parameter defined in Eq. (76)
$C_F(s)$	Fresnel cosine integral (evaluated at the point s)
c	adimensional parameter defined in Eq. (77)
c_k	adimensional constants defined in Eqs. (64)–(66) ($k=1,2,3$)
d	adimensional parameter defined in Eq. (78)
$Erf(s)$	Gauss error function
$\underline{e}$	unit vector along the axis of symmetry of the spacecraft
$F(t)$	function defined in Eq. (95)
$f(t)$	function defined in Eq. (16)
$G(z, c_k)$	function defined in Eq. (59)
g	parameter defined in Eq. (67)
$\underline{h}$	angular momentum vector of the spacecraft, N m s
I	moment of inertia of the spacecraft about the axis of symmetry, kg m^2
I_j	moments of inertia of the spacecraft about the principal axes ($j=1,2,3$), kg m^2
i	imaginary unit
k_j	parameter defined in Eqs. (85)–(92)
$\underline{M}$	external torque vector, N m
M_3	component of $\underline{M}$ along the third axis of B, N m
N	inertial Cartesian frame
(p,q,r)	components of $\underline{\omega}$ along the axes of B, rad/s
(p_0,q_0,r_0)	initial components of $\underline{\omega}$ along the axes of B, rad/s
(p_e,q_e,r_e)	components of $\underline{\omega}_e$ along the axes of B, rad/s
(p_h,q_h,r_h)	components of $\underline{\omega}_h$ along the axes of S, rad/s
p_0^*	parameter defined in Eq. (101), rad/s
q_0^*	parameter defined in Eq. (102), rad/s
R_{12}	direction cosine matrix, transposed of the rotation matrix, from the frame 2 to the frame 1
$R_{12}[i,j]$	element of the ith row and jth column of the direction cosine matrix R_{12}
r_0'	initial angular velocity component of the “virtual” sphere along the third axis of S, rad/s
r_{kj}	element kj of the direction cosine matrix R_{NK}, $k,j=1,2,3$
S	principal body-fixed Cartesian frame of the “virtual” spherical body with inertia I
$S_F(s)$	Fresnel cosine integral (evaluated at the point s)
t	time, s
t_F	final time, s
U	constant defined in Eq. (15), rad/s^2
w_k	stereographic complex rotation variable ($k=1,2,3$)
(x,y,z)	axis of the frame B
z	parameter defined in Eq. (60), rad$^{1/2}$
θ_j	Euler angles (sequence 1–2–3) from frame N to frame B ($j=1,2,3$), rad
${}^{s1}\theta_j\ (t)$	j-th Euler angle evaluated at the instant of time t with the solution s 1 ($j=1,2,3$), rad
θ^*	parameter defined in Eq. (30), rad
θ'	parameter defined in Eq. (106), rad
$\Omega\ (\omega)$	skew-symmetric matrix of the spacecraft angular velocity components, rad/s
$\underline{\omega}$	angular velocity vector of the spacecraft with respect to the inertial frame, rad/s
ω	$[p,q,r]^T$, rad
$\underline{\omega}_e$	vectorial component of $\underline{\omega}$ parallel to the axis of symmetry of the spacecraft, rad/s
ω_e	$[p_e,q_e,r_e]^T$, rad
$\underline{\omega}_h$	vectorial component of $\underline{\omega}$ parallel to $\underline{h}$, rad/s
ω_h	$[p_h,q_h,r_h]^T$
υ	parameter defined in Eq. (61), rad
${}_1F_1$	confluent hypergeometric function
${}^{s1,s2}E_j$	cumulative error of the solution s 1 with respect to the solution s 2 for the angle θ_j ($j=1,2,3$), rad
$(\bullet)_k$	Pochammer symbol

Subscript

B	spacecraft body-fixed frame
N	inertial frame
S	spherical body fixed frame
$\underline{\bullet}$	Gibbsian vector

Superscript

SB	approximated solution introduced by Wie
E	exact analytic solution
NA	new approximated solution obtained from the exact one
NI	numerical integration of the equations of motion
$\bar{\bullet}$	complex conjugate
$\underline{\dot{\bullet}}$	time derivative of a vector as evaluated in an inertial frame
$\dot{\bullet}$	time derivative of a scalar

the expressions for angular velocity, Euler angles, angular momentum, transverse and axial velocities, and transverse and axial displacements. Livneh and Wie [9] introduce nondimensional equations of motion for the spin-up maneuver of a generic semirigid body. The dynamics of an axially symmetric dual-spin spacecraft composed of two rigid bodies is studied by Hall [10]. Ayoubi and Goodarzi [11] formulate the equations of motion for the particular case of spinning spacecraft with sloshing effects. Oh [12] and Parkinson and Lange [13] propose