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Hybrid methods for determining time-optimal, constrained spacecraft reorientation maneuvers

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ABSTRACT

Time-optimal spacecraft slewing maneuvers with path constraints are difficult to compute even with direct methods. This paper examines the use of a hybrid, two-stage approach, in which a heuristic method provides a rough estimate of the solution, which then serves as the input to a pseudospectral optimizer. Three heuristic methods are examined for the first stage: particle swarm optimization (PSO), differential evolution (DE), and bacteria foraging optimization (BFO). In this two-stage method, the PSO-pseudospectral combination is approximately three times faster than the pseudospectral method alone, and the BFO-pseudospectral combination is approximately four times faster; however, the DE does not produce an initial estimate that reduces total computation time.

1. Introduction

The problem of reorienting a spacecraft in minimum time, often through large angles (so-called slew maneuvers) and subject to various constraints, can take a number of forms. For example, the axis normal to the solar panels may be required to lie always within some specified minimum angular distance from the sun-line. In cases where the control authority is low and the slew requires a relatively long time, certain faces of the vehicle may benefit from being kept as far as possible from the sun-line to avoid excessive solar heating. For scientific missions, observational instruments frequently must be kept beyond a specified minimum angular distance from high-intensity light sources to prevent damage.

Before addressing the time-optimal, constrained problem, it is useful to review what is known about the unconstrained problem. In a seminal paper, Bilimoria and Wie [1] considered time-optimal slews for a rigid spacecraft whose mass distribution is spherically symmetric, and which has equal control-torque authority for all three axes. Despite the symmetry of the system, the intuitively obvious time-optimal solution is not a λ–rotation (rotation through a specified angle about an axis fixed in both the body and an external reference frame) about the eigenaxis. Indeed, the fallacy here is that one easily confuses the minimum-angle of rotation problem (i.e., the angle about the eigenaxis) with the minimum-time problem, forgetting the constraints imposed by Euler's equations of rigid-body motion. Bilimoria and Wie found that the time-optimal solution includes precessional motion to achieve a lower time (approximately 10% less) than that obtained with an eigenaxis maneuver. Further, they found that the control history is bang-bang, with a switching structure that changes depending upon the magnitude of the angular maneuver (referring here to the final orientation in terms of a fictitious λ–rotation, with associated angle θ): for values of θ less than 72 degrees, the control history was found to contain seven switches between directions of the control torque components; larger values of θ require only five switches.

Several subsequent papers revisited the unconstrained problem, including such modifications as axisymmetric mass distribution and only two-axis control [2], asymmetric mass distribution [3], small reorientation angles [4], and combined time and fuel optimization [5]. Recently, Bai and Junkins [6] reconsidered the original problem (spherically symmetric

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mass distribution, three equal control torques) and found that at least two locally optimum solutions exist for reorientations of less than 72 deg. (one of which requires only six switches) if the controls are independently limited. Further, they proved that if the total control vector is constrained to have a maximum magnitude (i.e., with the orthogonal control components not necessarily independent), then the time-optimal solution is the eigenaxis maneuver.

Early work on constrained maneuvers focused upon generating feasible solutions [7,8] but did not attempt to find optimal solutions. The advent of pseudospectral methods and their application to various trajectory optimization problems has made it possible to study problems with challenging constraints [9]. These methods are not fully automatic and may require an intelligent initial guess. The method proposed in this paper is to use a two-step hybrid approach to achieve overall faster performance.

This work uses the Swift spacecraft [10] as an example of a vehicle that must be rapidly reoriented to align two telescopes at a desired astronomical target, namely, a gamma-ray burst. The satellite's Burst Alert Telescope (wide field of view) first detects the gamma-ray burst and the spacecraft then must reorient to allow the x-ray and UV/optical instruments to capture the rapidly fading afterglow of the event. To prevent damage to the these instruments, the slewing motion is constrained to prevent the telescopes' axis from entering established "keep-out" zones, defined as cones with central axes pointing to the Sun, Earth and Moon, with specified half-angles.

2. Problem statement

The problem is formulated as a Mayer optimal control problem, with performance index

$$J = t_f \tag{1}$$

where t_f is the final time to be minimized. Euler's equations of rigid-body motion describe the system dynamics

$$\begin{aligned} \dot{\omega}_1 &= [M_1 - \omega_2\omega_3(I_3 - I_2)]/I_1 \\ \dot{\omega}_2 &= [M_2 - \omega_3\omega_1(I_1 - I_3)]/I_2 \\ \dot{\omega}_3 &= [M_3 - \omega_1\omega_2(I_2 - I_1)]/I_3 \end{aligned} \tag{2}$$

These must be augmented with kinematic relationships in order to determine the orientation of the body over time. In this work, a formulation that uses Euler parameters is employed, with the relationship between the Euler parameters ϵ_i and the angular velocity components ω_j given by

$$\begin{bmatrix} \dot{\epsilon}_1 \\ \dot{\epsilon}_2 \\ \dot{\epsilon}_3 \\ \dot{\epsilon}_4 \end{bmatrix} = \begin{bmatrix} \epsilon_4 & -\epsilon_3 & \epsilon_2 & \epsilon_1 \\ \epsilon_3 & \epsilon_4 & -\epsilon_1 & \epsilon_2 \\ -\epsilon_2 & \epsilon_1 & \epsilon_4 & \epsilon_3 \\ -\epsilon_1 & -\epsilon_2 & -\epsilon_3 & \epsilon_4 \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ 0 \end{bmatrix} \tag{3}$$

The problem to be considered is a rest-to-rest maneuver, consisting of a rotation about the z-axis through an angle of $3\pi/4$ radians with boundary conditions

$$\begin{aligned} &\omega_1(0) = \omega_2(0) = \omega_3(0) = 0 \\ &\epsilon_1(0) = \epsilon_2(0) = \epsilon_3(0) = 0, \quad \epsilon_4(0) = 1 \\ &\omega_1(t_f) = \omega_2(t_f) = \omega_3(t_f) = 0 \\ &\epsilon_1(t_f) = \epsilon_2(t_f) = 0, \quad \epsilon_3(t_f) = \sin\left(\frac{3\pi}{8}\right), \\ &\epsilon_4(t_f) = \cos\left(\frac{3\pi}{8}\right) \end{aligned} \tag{4}$$

In the numerical calculations, time is scaled by the factor $\sqrt{I/M_{\max}}$ and the control torques are scaled by the factor $M_{\max}$.

For the optimal control problem formulated using Eqs. (1)–(3), the Hamiltonian is

$$\begin{aligned} \mathcal{H} = &\lambda_{\omega_1}\dot{\omega}_1 + \lambda_{\omega_1}\dot{\omega}_2 + \lambda_{\omega_3}\dot{\omega}_3 + \lambda_{\epsilon_1}\dot{\epsilon}_1 \\ &+ \lambda_{\epsilon_2}\dot{\epsilon}_2 + \lambda_{\epsilon_3}\dot{\epsilon}_3 + \lambda_{\epsilon_4}\dot{\epsilon}_4 \end{aligned} \tag{5}$$

For spacecraft of the type being modeled here (Swift, or other astronomical missions), one or more sensors is fixed to the spacecraft bus; these sensors all have the same central axis for their fields of view and this axis is designated here with the unit vector $\hat{\sigma}$ and referred to as the sensor axis. This axis must be kept at least a minimum angular distance α_x from each of several high-intensity light sources. Denoting the directions to these sources as $\hat{\sigma}_x$, where the subscript x can be S (Sun), E (Earth), or M (Moon), the so-called keep-out constraints are then written as

$$C_x = \hat{\sigma} \cdot \hat{\sigma}_x - \cos(\alpha_x) \le 0 \tag{6}$$

Without loss of generality, $\hat{\sigma}$ is assumed to lie along the body-fixed x-axis and its orientation with respect to the inertial frame is then determined [11, pp. 4–15]

$$\begin{aligned} \sigma_1 &= 1 - 2(\epsilon_1^2 + \epsilon_3^2) \\ \sigma_2 &= 2(\epsilon_1\epsilon_2 + \epsilon_3\epsilon_4) \\ \sigma_3 &= 2(\epsilon_3\epsilon_1 - \epsilon_2\epsilon_4) \end{aligned} \tag{7}$$

It is further assumed that the reorientation maneuver can occur quickly enough that the spacecraft's orbital position remains essentially unchanged, and that therefore, the inertial directions to the high-intensity sources also remain constant during the slew maneuver.

Inclusion of the path constraints, Eq. (6), requires augmenting the Hamiltonian with terms of the form $\mu_x C_x$, with the multiplier $\mu_x \ge 0$ if the constraint is active and $\mu_x = 0$ otherwise. While this problem could, in principle, be solved using an indirect method, the path constraints, Eq. (6), create especially difficult challenges. A direct approach, such as a pseudospectral method, offers the potential advantage of easier problem formulation; however, for this problem, an optimal solution, or even a feasible solution, is not automatically guaranteed.

To understand this challenge, one must recognize that the direct methods all employ an implicit integration of the governing system equations. Such solutions require initial estimates (sometimes these are just outright guesses) of the states and controls at the discrete times (nodes) for which the overall solution is computed. Lacking any user-provided initial estimate of the states and controls at these nodes, the logical automated estimate for the states is simply a linear interpolation between the states at the initial and final times; the control is also assumed to be a linear function between the minimum and maximum specified values, mapped over the specified range of times. Such a linear relationship violates various dynamic and kinematic constraints, viz. Eqs. (2) and (3). Consequently, the direct-method solver must then expend