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Nonlinear filtering methods for spacecraft navigation based on differential algebra

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ABSTRACT

The paper investigates the problem of nonlinear filtering applied to spacecraft navigation. Differential algebraic (DA) techniques are proposed as a valuable tool to implement the higher-order numerical and analytic extended Kalman filters. Working in the DA framework allows us to consistently reduce the required computational effort without losing accuracy. The performance of the proposed filters is assessed on different orbit determination problems with realistic orbit uncertainties. The case of nonlinear measurements is also considered. Numerical simulations show the good performance of the filter in case of both complex dynamics and highly nonlinear measurement problems.

1. Introduction

The nonlinear filtering problem plays an important role in various space-related applications and especially in orbit determination and spacecraft navigation problems. Near future sample and return missions from small bodies, landing missions to the Moon, Mars and outer planets as well as interplanetary exploration missions demand navigation systems based on accurate filtering techniques that are able to perform accurate trajectory estimation in a very reduced lapse of time.

At the present time the extended Kalman filter [1,2] (EKF) is mainly used for trajectory estimation. The EKF is based on the main idea of linearizing the equations of motion and the measurement equations via first-order Taylor expansions around the current mean and covariance. In some cases, however, the linear assumption fails to provide an accurate realization of the local trajectory motion due to the low frequency of the estimation process as well as the nature or the limited number of measurements. In such cases, a different method that accounts for the system nonlinearity must be used. An alternative approach is the unscented Kalman filter (UKF) [3,4] that yields superior performance with respect to the EKF in highly nonlinear situations because it is based on the unscented transformation, which does not contain any linearization. Even if the asymptotic complexity of the UKF algorithm is the same as for the EKF, in practice, the UKF is often slightly slower than the EKF. In 2007 Park and Scheeres [5,6] developed two nonlinear filters – the higher-order numerical extended Kalman filter (HNEKF) and the higher-order analytic extended Kalman filter (HAEKF) – by implementing a semi-analytic orbit uncertainty propagation technique, that is by solving for the higher-order Taylor series terms that describe the localized nonlinear motion and by analytically mapping the initial uncertainties. These higher-order filters are more accurate than the EKF, but the need to derive the so-called higher-order tensors makes them in many cases – especially for a sophisticated, high fidelity system model – difficult to use due to computational complexity. Due to this critical problem, up to now the HNEKF and the HAEKF have mainly been applied to the case of linear measurements. Up to now limited work has been done to automate and speed up the derivation of the state transition tensors [7,8].

Differential algebraic (DA) techniques are here proposed as a valuable tool to implement the HNEKF and the HAEKF, in order to obtain not only a higher-order filter, but also

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a computationally efficient one. Differential algebra supplies the tools to compute the derivatives of functions within a computer environment [9–12]. More specifically, by substituting the classical implementation of real algebra with the implementation of a new algebra of Taylor polynomials, any function f of n variables is expanded into its Taylor polynomial up to an arbitrarily order m. This has a strong impact when the numerical integration of an ordinary differential equation (ODE) is performed by means of an arbitrary integration scheme. Any integration scheme is based on algebraic operations, involving the evaluation of the ODE right hand side at several integration points. Therefore, starting from the DA representation of the initial conditions and carrying out all the evaluations in the DA framework, the flow of an ODE is obtained at each step as its Taylor expansion in the initial conditions. The accuracy of the Taylor expansion can be kept arbitrarily high by adjusting the expansion order. So, in the DA-based HNEKF and the DA-based HAEKF presented in this paper, both the propagation of the mean trajectory and the measurement function evaluation are carried out in the DA framework. The obtained solution map not only provides the pointwise values for the propagated state and measurements, but also provides the higher-order partials of the solution flow and of the measurement equation. This eliminates the need to calculate the higher-order tensors at each time step by solving a complex system of augmented ODE.

The proposed filters are tested on different orbit determination problems. First of all, a Sun–Earth halo orbit around the L1 point is considered to demonstrate the precise correspondence between our results and those obtained by Park and Scheeres with the original form of the HNEKF and HAEKF. Moreover, the case of an Earth orbiting satellite with realistic orbit uncertainties and nonlinear measurements is presented. Higher orders can improve the accuracy of the state determination since they can extract, from the available nonlinear measurements, more accurate information about the state of the vehicle than low order filters. Hence, numerical simulations show good performance of the filter in case of both complex dynamics and highly nonlinear measurement problems.

The paper is organized as follows. First an introduction to differential algebra and some hints on how to obtain high order expansion of the flow are presented. Then, after a brief overview about the higher-order extended Kalman filters, differential algebra is used to improve the performance of higher-order filters with respect to the original theory. Finally, the effectiveness of the method is demonstrated through numerical examples.

2. Notes on differential algebra

DA techniques, exploited here to obtain mth order Taylor expansions of the flow of a set of ODE's with respect to initial condition, were devised to attempt solving analytical problems through an algebraic approach [10]. Historically, the treatment of functions in numerics has been based on the treatment of numbers, and the classical numerical algorithms are based on the mere evaluation of functions at specific points. DA techniques rely on the observation that it is possible to extract more information on a function rather than its mere values. The basic idea is to bring the treatment of functions and the operations on them to computer environment in a similar manner as the treatment of real numbers. Referring to Fig. 1, consider two real numbers a and b. Their transformation into the floating point representation, $\overline{a}$ and $\overline{b}$ respectively, is performed to operate on them in a computer environment. Then, given any operation $*$ in the set of real numbers, an adjoint operation $\circledast$ is defined in the set of floating point (FP) numbers so that the diagram in Fig. 1 commutes. (The diagram commutes approximately in practice due to truncation errors.) Consequently, transforming the real numbers a and b into their FP representation and operating on them in the set of FP numbers returns the same result as carrying out the operation in the set of real numbers and then transforming the achieved result in its FP representation. In a similar way, let us suppose two m differentiable functions f and g in n variables are given. In the framework of differential algebra, the computer operates on them using their mth order Taylor expansions, F and G respectively. Therefore, the transformation of real numbers in their FP representation is now substituted by the extraction of the mth order Taylor expansions of f and g. For each operation in the space of m differentiable functions, an adjoint operation in the space of Taylor polynomials is defined so that the corresponding diagram commutes; i.e., extracting the Taylor expansions of f and g and operating on them in the space of Taylor polynomials (labeled as ${}_mD_n$) returns the same result as operating on f and g in the original space and then extracting the Taylor expansion of the resulting function. The straightforward implementation of differential algebra in a computer allows computation of the Taylor coefficients of a function up to a specified order m, along with the function evaluation, with a fixed amount of effort. The Taylor coefficients of order n for sums and products of functions, as well as scalar products with reals, can be computed from those of summands and factors; therefore, the set of equivalence classes of functions can be endowed with well-defined operations, leading to the so-called truncated power series algebra [13,14]. Similarly to the algorithms for floating point arithmetic, the algorithms for functions followed, including methods to perform composition of functions, to invert them, to solve nonlinear systems explicitly, and to treat common elementary functions [9,10]. In addition to these algebraic operations, the DA framework is endowed with differentiation and integration operators, therefore finalizing the definition of the DA structure.

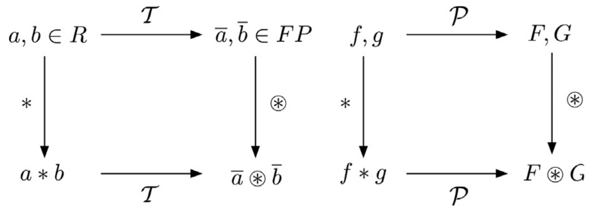

Fig. 1. Analogy between the floating point representation of real numbers in a computer environment (left figure) and the introduction of the algebra of Taylor polynomials in the differential algebraic framework (right figure).

2.1. The minimal differential algebra

Consider all ordered pairs (q_0, q_1), with q_0 and q_1 real numbers. Define addition, scalar multiplication, and vector