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An efficient harmonic balance method for unsteady flows in cascades ☆

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ABSTRACT

In this paper, nonlinear unsteady flows in cascades arising from self-excited blade vibrations are investigated using a mixed time/frequency domain harmonic balance technique. The time-periodic flow is computed at several sub-time levels that are equally spaced over a single period. The time derivative term in the unsteady Navier–Stokes equations is approximated by a pseudo-spectral operator, which couples solutions at different sub-time levels. Compared to the classical time-accurate approach, the present technique enjoys an improved computational efficiency. In addition, using complex periodic boundary conditions, the computational domain can be reduced to a grid spanning only a single blade in the cascade, which further improves the computational efficiency. With the use of the pseudo-spectral operator and pseudo-time marching, the problem can be treated as steady state and convergence acceleration techniques such as local time stepping, implicit residual smoothing and multigrid can be used. To demonstrate the efficiency and accuracy of the technique, we present results for the Tenth Standard and the Eleventh Standard Configurations.

1. Introduction

Accurate and efficient prediction of unsteady flows in turbomachinery is important since such predictions can improve the fundamental understanding of complex flow physics and reduce development time. In the literature to date, researchers have used three families of computational methods; the time-linearized method [10,4], the nonlinear time-accurate method [13,3] and the nonlinear harmonic balance method [14,9,11,5] to investigate unsteady flow phenomena in turbomachinery. In the time-linearized method the unsteady disturbances, which are harmonic in time, are assumed to be small compared to the mean flow variables. This assumption decouples the nonlinear unsteady flow into a nonlinear mean part and a linear small disturbance part. The resulting time-linearized equations can be solved very efficiently. However, because of the small disturbance assumption, these methods cannot model dynamically nonlinear problems, which may be important for many flows of interest. Unlike time-linearized methods, time-accurate methods do not rely on small disturbance and time-periodicity assumptions thereby allowing one to analyze nonlinear flow problems in a straight-forward manner albeit at increased computational cost, which can be on the order of 10 to 100 times the cost of typical time-linearized solvers.

In the past decade, the demand for efficient prediction of nonlinear unsteady flows led to the development of nonlinear frequency domain techniques. He and Ning [14] developed a nonlinear harmonic method to analyze unsteady flows in turbomachinery. In their approach they coupled the time-averaged flow with first-order harmonic disturbances through deterministic stresses. A more systematic and more generalized harmonic balance method was proposed by Hall et al. [9,11] for the analysis of nonlinear unsteady flows in cascades. In his approach, Hall [11] developed a mixed time-domain/frequency-domain technique where he computed dependent variables of the flowfield at sub-time levels equally spaced over a single period. The main advantages of the harmonic balance approach compared to a classical time-accurate method are its computational efficiency and its ability to model more accurately certain parts of the unsteady flow problem. For instance, the treatment of both the far-field and periodic boundary conditions are greatly simplified in the frequency domain. Complex periodicity conditions allow one to reduce the computational domain to a single blade passage in each row of a turbomachine, greatly reducing the computational cost. Furthermore, the use of a pseudo-spectral operator allows the unsteady problem to be treated as a number of coupled steady-state problems, which are computed simultaneously. According to our experience, the computational cost of the harmonic balance technique scales linearly with the number of sub-time levels for a relatively low reduced frequency and when the number of harmonics retained in the model is fewer than ten. Finally, convergence acceleration techniques originally designed for steady state computation may also be applied to this unsteady harmonic balance solver. In this

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paper, we use the mixed time-domain/frequency-domain harmonic balance technique to investigate unsteady flows arising from self-excited vibrations in turbomachinery and compare numerical solutions to available experimental data as well as computations of other researchers. Although the nonlinear harmonic balance technique has been used for various computational fluid dynamic (CFD) problems in the literature, only a few papers compared results to experimental data, which is one of the focal points of this paper.

2. Governing equations

To motivate the present method, consider two-dimensional Reynolds-averaged Navier–Stokes equations, with the Spalart–Allmaras [21] turbulence model. In strong conservation form, these equations are given by

$$\frac{\mathrm{d}}{\mathrm{d}t}\iint_{\mathcal{A}} \boldsymbol{U}\,\mathrm{d}\mathcal{A} + \int_{\mathcal{S}} [\boldsymbol{F},\boldsymbol{G}]\cdot\boldsymbol{n}\,\mathrm{d}\mathcal{S} = \iint_{\mathcal{A}} \boldsymbol{S}\,\mathrm{d}\mathcal{A} \tag{1}$$

where $\boldsymbol{U}$ is the vector of conservation variables, i.e.,

$$\boldsymbol{U} = \begin{Bmatrix} \rho \\ \rho u \\ \rho v \\ \rho E \\ \rho\tilde{\nu} \end{Bmatrix}$$

The last entry $\tilde{\nu}$, is the working variable in the Spalart–Allmaras turbulence model, from which the eddy viscosity is computed.

The flux vectors $\boldsymbol{F}, \boldsymbol{G}$ and the source term $\boldsymbol{S}$ are given by

$$\boldsymbol{F} = \begin{Bmatrix} \rho u - \rho\dot{f} \\ \rho u^2 + p - \tau_{xx} - \rho u\dot{f} \\ \rho uv - \tau_{xy} - \rho v\dot{f} \\ \rho uh - \tau_{xh} - \rho E\dot{f} \\ \rho u\tilde{\nu} - \tau_{x\nu} - \rho\tilde{\nu}\dot{f} \end{Bmatrix}$$

$$\boldsymbol{G} = \begin{Bmatrix} \rho v - \rho\dot{g} \\ \rho uv - \tau_{yx} - \rho u\dot{g} \\ \rho v^2 + p - \tau_{yy} - \rho v\dot{g} \\ \rho vh - \tau_{yh} - \rho E\dot{g} \\ \rho v\tilde{\nu} - \tau_{y\nu} - \rho\tilde{\nu}\dot{g} \end{Bmatrix}, \qquad \boldsymbol{S} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ S_t \end{Bmatrix}$$

where $\dot{f}$ and $\dot{g}$ are the x and y components of the unsteady grid motion velocity and τ_{xx} and τ_{xy} are the shear stresses defined as

$$\tau_{xx} = \frac{2}{3}(\mu_l + \mu_t)\left(2\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}\right)$$

$$\tau_{xy} = (\mu_l + \mu_t)\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$$

where the laminar viscosity μ_l is determined by Sutherland's law, and turbulence viscosity μ_t is determined by the Spalart–Allmaras turbulence model [21]. In addition, τ_{xh} term in the energy equation and $\tau_{x\nu}$ term in turbulence model equation are given by

$$\tau_{xh} = u\tau_{xx} + v\tau_{xy} + \left(\frac{\mu_l}{Pr_l} + \frac{\mu_t}{Pr_t}\right)\frac{\partial h}{\partial x}$$

$$\tau_{x\nu} = \frac{3}{2}(\mu_l + \rho\tilde{\nu})\frac{\partial\tilde{\nu}}{\partial x}$$

The remaining components of the shear stresses and terms in energy and turbulence model equations are defined similarly. For an ideal gas with a constant specific heat ratio, the pressure is related to the conserved variables through

$$p = (\gamma - 1)\rho\left[E - \frac{1}{2}\left(u^2 + v^2\right)\right]$$

Finally, the total enthalpy is defined as

$$h = \frac{\rho E + p}{\rho}$$

It should be noted that the inviscid flux and source vectors depend on the conservation variables and the Cartesian coordinates. The viscous fluxes depend on the gradients of the flow velocities, temperature and the working turbulent variable.

3. Harmonic balance approach

Here, it is assumed that the cascade blades vibrate harmonically with a frequency ω. The computational grid deforms to conform to this motion, so the grid also vibrates with a frequency ω about its mean position. Because the flow is temporally periodic, the flow variables may be represented as a Fourier series in time with spatially varying coefficients. For example, the conservation variables may be approximated as a truncated Fourier series given by

$$\boldsymbol{U}^*(x,y,t_i) = \boldsymbol{A}_0(x,y) + \sum_{n=1}^{N}\left[\boldsymbol{A}_n(x,y)\cos(\omega n t_i) + \boldsymbol{B}_n(x,y)\sin(\omega n t_i)\right]; \quad i = 1\text{–}(2N+1) \tag{2}$$

where ω is the fundamental excitation frequency, and $\boldsymbol{A}_0$, $\boldsymbol{A}_n$ and $\boldsymbol{B}_n$ are the Fourier coefficients of the conservation variables. In principle, the choice of N depends on the physical problem considered. For example, a single harmonic is generally sufficient for linear unsteady flows (such as those arising from low amplitude vibrations). On the other hand, more harmonics are required for strongly nonlinear unsteady flows. In such cases, a mode convergence study is usually required to ensure accuracy and determine an appropriate N. We must also note that in turbomachinery aeromechanics, it is often important to predict accurately only one or a very few of the Fourier coefficients of the unsteady flow. For instance, if one needed to calculate the forced response of a blade row at a natural frequency near the first harmonic, then it is only the first harmonic of the unsteady pressure (or more precisely the generalized force) that is important. In such cases where the lower harmonic content is of interest only a few harmonics are adequate to accurately predict those lower harmonics [6].

Note that the flow variables are computed and stored at $2N+1$ equally spaced points over one temporal period. Following Eq. (2), the Fourier coefficients can be determined from the sub-time level solutions by a discrete Fourier transform, i.e.,

$$\tilde{\boldsymbol{U}} = \mathbf{E}\boldsymbol{U}^* \tag{3}$$

Conversely, the conservation variables at the sub-time levels can be determined from the Fourier coefficients by the inverse discrete Fourier transform given by

$$\boldsymbol{U}^* = \mathbf{E}^{-1}\tilde{\boldsymbol{U}} \tag{4}$$

Note that $\mathbf{E}$ and $\mathbf{E}^{-1}$ are square matrices as the number of sub-time levels is equal to the number of Fourier coefficients. More specifically, $\mathbf{E}$ and $\mathbf{E}^{-1}$ are given as