Available online at www.sciencedirect.com
SCIENCE DIRECT
Aerospace Science and Technology 9 (2005) 626–634
Aerospace Science and Technology
www.elsevier.com/locate/aescte

A new steering law for redundant control moment gyroscope clusters

Ozan Tekinalp *, Emre Yavuzoglu

Middle East Technical University, Aerospace Engineering Department, 06531 Ankara, Turkey

Received 26 August 2004; received in revised form 27 May 2005; accepted 7 June 2005
Available online 14 July 2005

Abstract

A new inverse kinematics algorithm is developed that may provide singularity avoidance or may be used for quick transition through a singularity with small torque errors. To avoid singularities, angular momentum trajectory of the control moment gyro cluster during the maneuver is to be simulated in advance for the calculation of singularity free gimbal histories. The steering law proposed accurately generates the required torques making it suitable to be used in a feedback system. The desired gimbal trajectories are also closely followed as long as the difference between the planned and requested angular momentum histories are close. A numbers of approaches suitable for the spontaneous response of the spacecraft for singularity avoidance or quick transition through singularities are proposed and presented as well. It is also shown that the computational requirements of the new steering law are comparable to the other commonly used steering laws.

Keywords: Control moment gyroscopes; Steering law; Inverse kinematics algorithm; Spacecraft attitude control

1. Introduction

Due to their superior properties such as large torque amplification and momentum storage, control moment gyro (CMG) based attitude control systems are very attractive for space applications. In fact such actuators have been used in a number of large spacecraft such as MIR, Skylab, and ISS [14].

For a specified torque level, single gimbal control moment gyros (SGCMG) based systems exhibit benefits in power requirements, agility, weight, and size over their competitors such as reaction and momentum wheels. Their construction is much simpler than double gimbal control moment gyros. Besides their many advantages, singularity problems make their use a real challenge. Thus, during large slew maneuvers, they may steer towards singular configurations, which allow no torque capability along a particular direction [14].

An early work of Jacot and Liska [4] investigates the use of control moment gyros for spacecraft attitude control. Using basic geometry, Margulies and Aubrun [6] discussed and established the fundamental properties of such clusters. They investigated the momentum envelope for various CMG configurations and identified the singular configurations. They also presented the possibilities of escaping from singular configuration by null motion for redundant systems. Bedrossian et al. [1,2], recognizing the similarities between robotic manipulators and CMG systems, utilized the singularity-robust inverse (SR-inverse) technique developed by Nakamura and Hanafusa [7] to obtain approximate solution of gimbal rates allowing some torque error in the vicinity of singularity. In addition they proposed to add null motion to the particular solution to avoid singularities. Oh and Vadali [8] provided complete set of equations of motion including the rotor transverse inertia as well as gimbal inertia terms. Using the Lyapunov's approach, they formulated an alternative feedback control law that employs gimbal acceleration steering instead of velocity steering. Krishnan and Vadali [5], again using Lyapunov's method developed an inverse free technique for spacecraft control. Wie et al. [15] by modifying the SR-inverse method introduced a new logic

* Corresponding author. Phone: +90 (312) 210 4287; fax: +90 (312) 210 1272.
E-mail address: tekinalp@metu.edu.tr (O. Tekinalp).

1270-9638/$ – see front matter
doi:10.1016/j.ast.2005.06.001

that helps the spacecraft transit through internal singularities.

Paradiso [9] presented a directed search algorithm, which is capable of globally avoiding singular states in an open loop fashion, utilizing null motion at discrete nodes. Vadali et al. [13] has shown that it is possible to find a set of preferred initial gimbal angles that would not encounter singularities during a particular maneuver. Vadali and Krishnan [12] worked on explicitly avoiding singularities by parameterizing gimbal rates as polynomial functions of time and optimizing the parameters with respect to a singularity avoidance objective function.

Newer space missions require accurate generation of autopilot requested torques during a maneuver. For example, tracking of a ground target for multiple imaging, for the construction of three-dimensional images, is one application that requires precise control of the spacecraft. These pointing and tracking requirements will be more stringent as the satellites with decimeter ground sampling distances emerge. Available online steering laws cannot avoid internal singularities, and usually require considerable amount of time to transit through singularities with large torque errors, leading attitude control errors. They also require large gimbal rates during this transit as well. The pre-planned techniques may successfully avoid singularities. However, the approaches that have been presented in the literature are all open loop approaches [9,12,13].

In this study a new inverse kinematics steering law is presented. The law can avoid internal singularities, by planning the maneuver in advance, and obtaining preferred gimbal trajectories to ride on. However, it is different from the preplanned approaches presented in the literature, since it can be used in a feedback scheme to accurately generate desired torques. The law may also be used for spontaneous steering either to avoid or rapidly transit through internal singularities.

In the next section fundamental equations for a spacecraft attitude control with an SGCMG cluster is presented. A summary of the available steering laws is followed by the derivation of the new steering law together with its singular value analysis. Manuscript continues with a number of simulations to show the effectiveness of the new steering law. Next the computational requirements of the new steering law are compared to the other laws available. Finally, conclusions are given.

2. Formulation of the problem

Total angular momentum of the spacecraft in the body fixed frame may be expressed as follows [14]:

$$\mathbf{H} = \mathbf{I}_{\mathrm{S}}\boldsymbol{\omega} + \mathbf{h}, \tag{1}$$

where $\mathbf{I}_{\mathrm{S}}$ is the inertia tensor, $\boldsymbol{\omega}$ is the angular velocity vector of the spacecraft, and $\mathbf{h}$ is the total angular momentum of the CMG cluster. According to the Newton's 2nd law, the rotational equations of motion of such a spacecraft may be written in the body fixed frame as:

$$\mathbf{I}_{\mathrm{S}}\dot{\boldsymbol{\omega}} + \boldsymbol{\omega} \times \mathbf{I}_{\mathrm{S}}\boldsymbol{\omega} = \mathbf{T}_{\mathrm{ext}} + \mathbf{u}, \tag{2}$$

$$\dot{\mathbf{h}} + \boldsymbol{\omega} \times \mathbf{h} = -\mathbf{u}, \tag{3}$$

where $\mathbf{T}_{\mathrm{ext}}$ is the sum of external disturbance torques acting on the spacecraft (i.e., gravity gradient torque, solar radiation pressure, etc.), and $\mathbf{u}$ is the internal control torque. Additional differential equations that relate body rates to the attitude parameters (i.e., quaternions, Euler angles, etc.) are also needed to describe the spacecraft attitude. The spacecraft control input, $\mathbf{u}$, will be generated by the CMG actuator cluster. A control law with feedback terms due to the attitude error in terms of quaternions and angular rate vectors may be used [14]:

$$\mathbf{u} = -\mathbf{K}\mathbf{q}_{\mathrm{e}} - \mathbf{D}\boldsymbol{\omega}. \tag{4}$$

In the above equation, $\mathbf{q}_{\mathrm{e}} = [q_{1\mathrm{e}}\ q_{2\mathrm{e}}\ q_{3\mathrm{e}}]^{\mathrm{T}}$ is the quaternion direction error vector between the desired quaternion and the current quaternion. The feedback gain matrices $\mathbf{K}$ and $\mathbf{D}$ shall be properly selected to realize the desired transient performance of the system [14].

Minimally redundant SGCMG clusters, containing rotors with equal and constant angular momentum are usually configured in a pyramid-mounting configuration [12–16]. Their total angular momentum, $\mathbf{h} = \mathbf{h}(\beta, \boldsymbol{\delta})$, is the function of the gimbal angles $\boldsymbol{\delta} = [\delta_1, \delta_2, \delta_3, \delta_4]^{\mathrm{T}}$, while β is the constant pyramid skew angle. In this manuscript $\beta = 54.75°$ is used since it gives a nearly spherical momentum envelope [9,14]. In addition, each gyro is assumed to have an angular momentum of unit magnitude (1 N m s). The total output torque of the CMG cluster is given by:

$$\boldsymbol{\tau} = \mathrm{d}\mathbf{h}/\mathrm{d}\mathbf{t} = [\partial\mathbf{h}/\partial\boldsymbol{\delta}]\dot{\boldsymbol{\delta}} = \mathbf{J}(\boldsymbol{\delta})\dot{\boldsymbol{\delta}}. \tag{5}$$

In this case, $\mathbf{J}$ is a (3×4) Jacobian matrix, since there are four CMGs. For a command, $\mathbf{u}$, generated by the feedback system (Eq. (4)), the torque requirement (i.e., $\boldsymbol{\tau}_{\mathrm{desired}} = -\boldsymbol{\omega} \times \mathbf{h} - \mathbf{u}$) is realized by riding the gimbals at proper rates obtained solving Eq. (5) simultaneously. For example, minimum two-norm solution of Eq. (5), gives the Moore–Penrose pseudo inverse [15]:

$$\dot{\boldsymbol{\delta}}_{\mathrm{MP}} = \mathbf{J}^{\mathrm{T}}\left[\mathbf{J}\mathbf{J}^{\mathrm{T}}\right]^{-1}\boldsymbol{\tau}_{\mathrm{desired}}. \tag{6}$$

Mathematically, this equation fails when $\mathbf{J}$ loses rank since inverse of $\mathbf{J}\mathbf{J}^{\mathrm{T}}$ cannot be taken. Physical interpretation is that if all output torque vectors remain on the same plane, no output torque can be produced along the direction normal to this plane, called the singularity direction [6]. Two main kinds of singularities may be identified. The singularity due to momentum saturation occurs when the angular momentum vector reaches its maximum possible value along any given direction. The remaining singularities are called internal singularities [9,14]. Consequently, one measure of singularity may be:

$$m = \det\left(\mathbf{J}\mathbf{J}^{\mathrm{T}}\right). \tag{7}$$