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A new design approach of *PD* controllers

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Abstract

A *PD* controller design problem for the system described by a second order dynamic model is investigated in this paper. A new *PD* type controller is proposed by means of Kharitonov theorem and Bilharz criteria. It is proved that using only two parameters is good enough to stabilize general second order dynamic system despite of its dimensions and coupling characteristics. Unlike the classical *PD* controller design techniques, this approach does not need to tune the design parameters frequently, so the design procedure is greatly simplified. Especially for the system with asymmetric parties in generalized damping matrix and generalized stiffness matrix, this method transfers the controller design for multivariable system to the choice of a scalar parameter μ. This simple and easy to implement approach can be used for the *PD* controller design of linear coupled multivariable systems. The attitude control of a spacecraft is used as an example in this study.

Keywords: Linear dynamic system; *PD* controller design; Bilharz criteria; Kharitonov theorem

1. Introduction

Since the advantages of being easy to realize, structurally simple, and robust, etc., *PD* and *PID* controllers have been widely used in various control fields, such as: industrial process control, chemical process control, spacecraft control, robot control, vehicle control and so on. The key of using *PD* and *PID* controllers is to tune their parameters. In the past several decades, a lot of approaches have been proposed for tuning the parameters of *PD* and *PID* controllers, for example, the pole placement approach, the LQR approach, and the self-tuning approach (see [2,6,8,16–18] and the references therein). However, it is not easy to tune the parameters of *PD* and *PID* controllers for multivariable control systems, especially for the coupled multivariable systems. The present paper concerns the *PD* controller design for the dynamic systems described by the second order differential equations. Here, the original problem for the coupled multivariable system is transferred to the problem for the single variable system. This idea comes from the stability theory of dynamic systems.

During the past four decades, the stability problem of the linear dynamic systems has been received much attention by the researchers working on both the fields of control theory and mechanics. Many research results have been reported (see [1,4,5,9–11,13–15,20–22] and the references therein). The linear dynamic system, generally, takes the form of the following second order differential equation:

$$M_0\ddot{q} + (D_0 + G)\dot{q} + (K_0 + C)q = F, \tag{1}$$

where, q is the generalized coordinate vector, M_0, D_0 and K_0 are the known generalized mass matrix, generalized damping matrix and generalized stiffness matrix respectively, G is the gyro coupling matrix, C is the cycle matrix. Generally, M_0, D_0 and K_0 are symmetric, G and C are skew-symmetric. F is the generalized applied force vector. If $F = 0$ the system is called autonomous, otherwise it is called non-autonomous. For the autonomous system, if $D_0 \neq 0$ or/and $C \neq 0$, the system is non-conservative, otherwise the system is conservative. For the non-autonomous system, F

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is generally the sum of the control force and the disturbance force, while F is chosen as *PD* control law. There are a lot of approaches to analyze the stability of the system (1), for example, the *KTC* theorem [4,14,22] and the extended *KTC* theorem [5,9,10]. These approaches require either checking the positive definite property or calculating the eigenvalues of relative matrices in Eq. (1). Considering the uncertainty of the coefficient matrices in dynamic systems, some research results about the robustness of dynamic systems have been reported [1,11,21]. The stability analyses based on matrix polynomial theory have also been proposed [13,15,20,21]. The aforementioned stability analyses are inconvenient for *PD* or *PID* controllers design.

This paper deals with *PD* controller design problem for the systems described by a second order dynamic model. A new *PD* typed controller is proposed by means of Kharitonov theorem and Bilharz criteria. The main contribution of this paper is transforming the original *PD* controller design problem for multivariable systems to that for a single variable system. Unlike the classical *PD* controller design techniques, this approach does not need to tune the design parameters frequently, so the design procedure is simplified. It is proved that using only two parameters is good enough to stabilize a second order dynamic system despite of its dimensions. Especially for the systems with skew-symmetric parties in the generalized damping matrix and the generalized stiffness matrix, only one scalar parameter μ needs to be chosen. An example for attitude control of spacecraft illustrates its validity.

2. Preliminaries

Hermitian decomposition. $\forall A \in C^{n\times n}$ can be decomposed as

$$A = B + jC \tag{2}$$

where

$$B = \frac{A + A^*}{2}, \quad C = \frac{A - A^*}{2j}, \quad j = \sqrt{-1}. \tag{3}$$

Obviously, B and C are Hermitian matrices.

Definition of value set [3]. Assume $A \in C^{n\times n}$, $v \in C^n$, the value set $V(A)$ of A is defined as

$$V(A) = \{v^* A v \mid \|v\| = 1\}. \tag{4}$$

Denoting the eigenvalues of B and C respectively as $\lambda_i(B)$ and $\lambda_i(C)$, and assuming that they are arranged by degressive order

$$\lambda_1(B) \leqslant \lambda_2(B) \leqslant \cdots \leqslant \lambda_n(B),$$

$$\lambda_1(C) \leqslant \lambda_2(C) \leqslant \cdots \leqslant \lambda_n(C)$$

one has the following lemma.

Lemma 1 [3]. *The value set of matrix A is located in the rectangle region of complex plane with vertices $(\lambda_1(B), j\lambda_1(C))$, $(\lambda_n(B), j\lambda_1(C))$, $(\lambda_n(B), j\lambda_n(C))$, $(\lambda_1(B), j\lambda_n(C))$.*

Consider the following polynomial with complex coefficients

$$P(s) = s^n + (a_{n-1} + jb_{n-1})s^{n-1} + (a_{n-2} + jb_{n-2})s^{n-2} + \cdots + (a_1 + jb_1)s + (a_0 + jb_0). \tag{5}$$

Lemma 2 (Bilharz criteria [12]). *The polynomial* (5) *is Hurwitz, if and only if all even order principal minors of its Bilharz matrix β are positive definite, where*

$$\beta \triangleq \begin{bmatrix} 1 & b_{n-1} & -a_{n-2} & -b_{n-3} & a_{n-4} & b_{n-5} & -a_{n-6} & -b_{n-7} & \cdots \\ 0 & a_{n-1} & b_{n-2} & -a_{n-3} & -b_{n-4} & a_{n-5} & b_{n-6} & -a_{n-7} & \cdots \\ 0 & 1 & b_{n-1} & -a_{n-2} & -b_{n-3} & a_{n-4} & b_{n-5} & -a_{n-6} & \cdots \\ 0 & 0 & a_{n-1} & b_{n-2} & -a_{n-3} & -b_{n-4} & a_{n-5} & b_{n-6} & \cdots \\ 0 & 0 & 1 & b_{n-1} & -a_{n-2} & -b_{n-3} & a_{n-4} & b_{n-5} & \cdots \\ 0 & 0 & 0 & a_{n-1} & b_{n-2} & -a_{n-3} & -b_{n-4} & a_{n-5} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix}. \tag{6}$$

For details see Ref. [12].

Definition 1. The polynomial (5) is called an internal polynomial if its coefficients satisfy

$$\underline{a}_i \leqslant a_i \leqslant \bar{a}_i, \quad \underline{b}_i \leqslant b_i \leqslant \bar{b}_i, \quad i = 0, 1, 2, \ldots, n-1, \tag{7}$$

where $\underline{a}_i, \bar{a}_i$ and $\underline{b}_i, \bar{b}_i$ represent the lower and upper bounds of a_i and b_i respectively.

Lemma 3 (Kharitonov theorem [3]). *The internal polynomial* (5) *is Hurwitz if and only if the following eight vertex polynomials are Hurwitz*:

$$P_1(s) = (\underline{a}_0 + j\underline{b}_0) + (\underline{a}_1 + j\bar{b}_1)s + (\bar{a}_2 + j\bar{b}_2)s^2 + (\bar{a}_3 + j\underline{b}_3)s^3 + \cdots, \tag{8a}$$

$$P_2(s) = (\bar{a}_0 + j\bar{b}_0) + (\bar{a}_1 + j\underline{b}_1)s + (\underline{a}_2 + j\underline{b}_2)s^2 + (\underline{a}_3 + j\bar{b}_3)s^3 + \cdots, \tag{8b}$$

$$P_3(s) = (\bar{a}_0 + j\underline{b}_0) + (\underline{a}_1 + j\underline{b}_1)s + (\underline{a}_2 + j\bar{b}_2)s^2 + (\bar{a}_3 + j\bar{b}_3)s^3 + \cdots, \tag{8c}$$

$$P_4(s) = (\underline{a}_0 + j\bar{b}_0) + (\bar{a}_1 + j\bar{b}_1)s + (\bar{a}_2 + j\underline{b}_2)s^2 + (\underline{a}_3 + j\underline{b}_3)s^3 + \cdots, \tag{8d}$$

$$P_5(s) = (\underline{a}_0 + j\underline{b}_0) + (\bar{a}_1 + j\underline{b}_1)s + (\bar{a}_2 + j\bar{b}_2)s^2 + (\underline{a}_3 + j\bar{b}_3)s^3 + \cdots, \tag{8e}$$

$$P_6(s) = (\bar{a}_0 + j\bar{b}_0) + (\underline{a}_1 + j\bar{b}_1)s + (\underline{a}_2 + j\underline{b}_2)s^2 + (\bar{a}_3 + j\underline{b}_3)s^3 + \cdots, \tag{8f}$$

$$P_7(s) = (\bar{a}_0 + j\underline{b}_0) + (\bar{a}_1 + j\bar{b}_1)s + (\underline{a}_2 + j\bar{b}_2)s^2 + (\underline{a}_3 + j\underline{b}_3)s^3 + \cdots, \tag{8g}$$

$$P_8(s) = (\underline{a}_0 + j\bar{b}_0) + (\underline{a}_1 + j\underline{b}_1)s + (\bar{a}_2 + j\underline{b}_2)s^2 + (\bar{a}_3 + j\bar{b}_3)s^3 + \cdots. \tag{8h}$$