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Exponential utility functions aid upstream decision making

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ABSTRACT

Although the concepts and mathematics of utility theory and its application to adjusting valuations to reflect the perspectives of decision makers with a range of risk preferences have been established for decades, these concepts and numerical applications remain relatively rarely applied by decision makers in the upstream gas and oil industries. Utility functions are now extensively used to assist evaluation of oil and gas hedging and trading of financial and physical commodities from the risk preferences of the parties involved. This study makes the case for more extensive use of utility functions in the upstream gas and oil sectors by presenting cases that highlight both the conceptual and valuation benefits that result from their application.

Exponential utility functions adequately describe the risk preferences of risk-averse and risk-prone decision makers for a wide range of upstream gas and oil asset types and circumstances. Simple equations for the calculation of utility factors and expected utility factors, i.e., taking into account probabilities of a range of outcomes being realised, are presented and compared with the equivalent linear utility functions of a risk-neutral investor valuing assets based on unrisked discounted cash flow (i.e. net present value, NPV) and risked discounted cash flow (i.e., expected monetary value, EMV). The additional insight gained from applying utility functions is considered with examples for high-uncertainty exploration assets, decision makers constrained by various loss tolerances and selection of optimum gas field development plans from a number of distinct alternative plans. In all cases considered the utility functions provide decision makers with greater insight than just the consideration of NPV and/or EMV. A case is therefore made to justify more extensive use of utility functions by upstream decision makers.

1. Introduction

Since the 1960s it has been established that investors' perceptions and attitudes towards uncertainty and risk can influence the way in which they value assets and make investment decisions upon them (e.g., Hammond, 1967; Swalm, 1966). This work built upon the earlier mathematical development of classical utility theory (Von Neumann and Morgenstern, 1944; Herstein and Milnor, 1953) that itself evolved from applying game theory to economic behaviour. The mathematical definition of various utility models and preference theory has continued to evolve (e.g. Hammond, 1974; Shepherdson, 1980; Starmer, 2000; Aliev et al., 2016).

Utility theory and quantifying risk preferences with respect to oil and gas exploration and production have been widely discussed in the general context of risk analysis approaches (e.g. Macmillan, 2000; Motta et al., 2000; Ozdogan, 2004; Suslick and Schiozer., 2004; Byrska-Rąpała, 2012). Studies focused on making decisions in the upstream oil and gas industry under conditions of uncertainty, characterized by vague and imprecise estimates of reserves and future production have also touched upon the concepts of risk preferences and utility theory (e.g., Bickel and Bratvold, 2008; Bratvold and Begg, 2008). Multi-attribute utility theory is also being applied in the decision analysis associated with the decommissioning of offshore oil and gas platform (Henrion et al., 2015). However, there remains considerable scope to expand the upstream applications of these tools.

Several non-linear definitions of utility theory are available and generally match observed behaviours of investors more realistically than linear models. Prospect theory, a non-expected utility theory (Kahneman and Tversky, 1979), which allows preferences for risky decisions to be nonlinear in both outcomes and probabilities, may be a relevant approach in reflecting public perception of assigning

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high utility to very low probability events such as certain rare but severe industrial accidents and severe environmental damage (i.e., spills and pollution) (Bartczak et al., 2015). Ignoring probability weightings can lead to the expected utility of certain decision makers being under-estimated in classic utility models (Riddel, 2012).

Consider an expected utility-maximizing decision maker who has the opportunity to hold interests in two assets and wishes to rank its preference to invest in one or other of the assets. Utility theory suggests that a better decision will be made if the assets are ranked in accordance with an objective to maximize their expected utility rather than in accordance with maximizing the expected discounted cash flow value, i.e., risk adjusted net present value NPV, and/or internal rates of return, IRR.

Utility-based decision support models are now quite extensively proposed and applied in oil and gas trading and hedging activities (e.g., Cotter and Hanly, 2012; Lean et al., 2015), but less so in the upstream sector. It is worth considering how relatively-easy-to-construct expected utility models can provide insight and assistance to upstream decision making for relatively little additional effort to classic risk-adjusted discounted cash flow analysis.

We examine how risk preferences and loss aversion affect decision makers' choices significantly across the various sectors of the upstream gas and oil industries, yet the industry often doggedly relies upon unrisked discounted cash flow analysis valuations (e.g. NPV and IRR), without attempting to translate such values into expected utilities to help further refine and rationalize their decisions. This study, therefore, makes a case for the inclusion of expected utility calculations to support investment decisions in the upstream gas and oil industry.

2. Basic concepts help to visualize risk preferences

An exponential utility function for NPV (net present value – a discounted cash flow value) is useful for explaining the risk preferences of oil and gas investment decision makers that are not indifferent to their risk exposure. One way to express an exponential utility function is to use the equation:

$$U(x) = \left[1 - e^{(-r*x)}\right] \Big/ 1 - e^{(-r)} \quad (r \neq 0) \tag{1}$$

Where, U(x) is the utility function between zero and 1 for NPV x also scaled/normalised to a zero to 1 scale; and r is a risk aversion factor $\neq 0$. This equation can also be expressed using the inverse of r, i.e. a risk tolerance factor, c, such that $c = 1/r$.

In Equation (1) as the value of r increases to more positive values the curved utility function become more convex (see Fig. 1; reflecting more risk-averse tendencies), whereas, as values of r decrease to more negative numbers the curved utility function become more concave (see Fig. 1; reflecting more risk-seeking tendencies).

If $r = 0$, implying that a decision-maker is indifferent to risk Equation (1) does not apply and the utility function of that risk neutral investor is depicted by the simple relationship:

$$U(x) = x \quad (r = 0) \tag{2}$$

Equations (1) and (2) are evaluated in Table 1 and Fig. 1 to illustrate in simple terms the utility functions of risk-averse, risk-prone and risk-neutral decision makers.

Fig. 1 describes three different types of risk reaction (tolerance) behaviour; each with an objective of making decisions that maximize value as it is perceived:

Risk neutral (r=0): a linear relationship with value (i.e., Equation (2)) indicating that the decision maker is ambivalent to risk and focused on value. The appeal of a certain asset increases linearly to such an investor based upon its net present value (NPV). The slope and intercept of the straight line could be adjusted by coefficients added to Equation (2), but the essential feature is that value and utility are related in a linear manner.

Risk averse (r is positive): the curves calculated from Equation (1) are convex in shape when viewed from the top left of Fig. 1; as r increases the risk-averse utility curves become more convex. Risk-

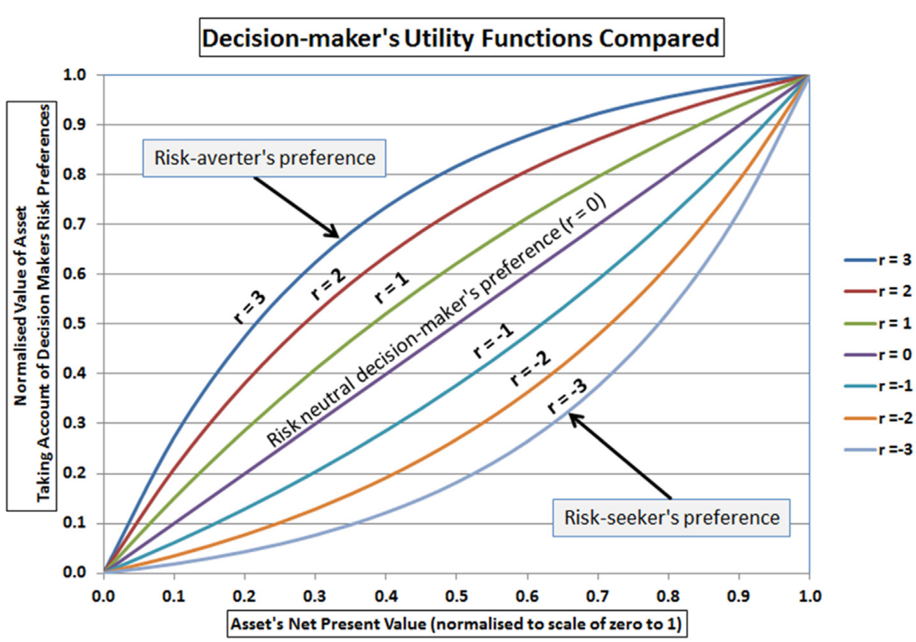

Fig. 1. Decision makers' non-linear utility function relationships to linear discounted cash flow valuations of an asset varying between zero and 1. This diagram expands upon the established concepts of exponential utility functions (Hammond, 1967; Guyaguler and Horne, 2004). The curves display the data included in Table 1 and are derived from Equations (1) And (2).

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