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Short Communication

Nonlinear frequency mixing in a resonant cavity: Numerical simulations in a bubbly liquid

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ABSTRACT

The study of nonlinear frequency mixing for acoustic standing waves in a resonator cavity is presented. Two high frequencies are mixed in a highly nonlinear bubbly liquid filled cavity that is resonant at the difference frequency. The analysis is carried out through numerical experiments, and both linear and nonlinear regimes are compared. The results show highly efficient generation of the difference frequency at high excitation amplitude. The large acoustic nonlinearity of the bubbly liquid that is responsible for the strong difference-frequency resonance also induces significant enhancement of the parametric frequency mixing effect to generate second harmonic of the difference frequency.

1. Introduction

Nonlinear interaction of two collinear high-frequency sounds beams propagating in a fluid results in the generation of both sum and difference frequency components due to mixing of these frequencies in the fluid [1]. Typically, the sum frequency gets absorbed in the fluid while the difference frequency continues. This low frequency signal is characterized by a narrow beam-width and is nearly side-lobe free, which is useful in many applications, such as sonar, depth sounding, sea-floor profiling, directional communication, and medical ultrasound. This nonlinear effect was described theoretically by Westervelt [2] who showed that the difference frequency wave could be considered to be radiated from an array of acoustic sources distributed continuously throughout the interaction volume similar to an end-fire array. This parametric end-fire array was first demonstrated experimentally by Bellin and Beyer [3] in water and later the work was expanded on by Berktay [4]. The development of underwater parametric sonar relies on this nonlinear interaction process.

Besides water, other nonlinear fluids (organic fluids) [5] and materials (rubber, granular media) [6,7] have been used for frequency mixing and difference frequency generation. Recently, Sinha and Pantea showed enhanced frequency mixing and difference frequency beam generation in Fluorinert [8].

The study of acoustical nonlinear behavior of liquid containing microbubbles has been of interest for many years. The increasing interest in bubbly liquids is related to the fact that existence of bubbles may enhance the acoustic nonlinearity of a liquid due to the nonlinear oscillations of bubbles even at very small void fractions [9], over a large frequency band covering their resonance and frequencies below. In a bubbly fluid, contributions to the bulk acoustic nonlinearity arise due to primary contributions from (i) nonlinearity of the equation of state of the liquid, (ii) nonlinearity of the equation of state of the gas, and (iii) the dynamical nonlinearity of the bubbles (dominant at bubble resonance and on a large frequency range below resonance).

If two primary acoustic waves of different frequencies are incident on a bubbly fluid, the scattered field will include both sum and difference frequencies and various harmonics [10]. Therefore, the increased nonlinearity can enhance the efficiency of parametric array (due to difference frequency generation in an end-fire array configuration) in underwater acoustics (e.g., communication and sea-bottom imaging) [11] and also for medical imaging (e.g., detection of gas bubbles in blood or tissue), and bubble sizing [12,13] in industrial applications. Harmonic imaging using ultrasound contrast agents (microbubbles) in medical ultrasound imaging has become an emerging field. The effective nonlinearity of bubbly liquids can be orders of magnitude greater at resonance and at

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frequencies below than at frequencies above. Kobelev and Sutin [14] conducted the first experimental demonstration of the frequency mixing in bubbly liquids. Woodsum [15] suggested the use of air bubbles to enhance the efficiency of parametric frequency mixing due to the large acoustic nonlinearity of such bubbles. In addition, such a system also provided significantly higher loss of the primary frequencies if the bubbles are resonant. Druzhinin et al. [16] showed a way to enhance mixing efficiency while reducing loss by using a layer of non-resonant bubbles. Even non-resonant bubbles have a much stronger nonlinearity than that of water and a bubble layer has a lower sound speed than water below bubble resonance. Such a bubble layer can also produce resonance effects due to reflecting boundaries and significantly enhance difference frequency generation.

Cavity resonant enhancement of nonlinear processes has been studied for several decades in the electromagnetic spectrum and in various frequency conversion devices. In comparison, such studies in acoustics, in particular, in nonlinear fluids is limited. Strong difference frequency generation in a small geometry such as in a resonator cavity has many potential applications in several areas of technology. Any fluid with acoustic nonlinearity can be used in a resonant cavity to enhance the resonance of the difference frequency produced by parametric frequency mixing.

This fact has motivated this study of enhanced difference frequency generation in a small cavity using bubbly fluid. Here, we present our first theoretical analysis to observe the magnification of the difference frequency component in a nonlinear fluid-filled resonant cavity. This analysis is carried out via numerical simulations. We plan to corroborate the results of this analysis through experiments in the near future.

This study focuses on numerical simulations made at off-resonance frequencies. The primary frequencies are selected to be below the bubble resonance frequency: the bubbly medium at both primary frequencies has very high acoustic nonlinearity.

In contrast to the study in Ref. [16], our study simply takes advantage of the nonlinearity of the bubbly fluid in the cavity and does not directly excite resonance in a bubbly layer. The differential model used in Ref. [16] is a quasistatic model for isothermal gas, dependent on the volume fraction of gas but independent of the bubble radius, while the differential system we use considers an adiabatic gas and assumes the bubble volume variation as an unknown variable of the problem, which allows the nonlinear dynamics of the bubbles to affect the standing wave, and vice versa. In addition, the study in Ref. [16] used frequencies much lower than this work with respect to the bubble resonance.

2. Materials and methods

We consider a mixture of water and a high-density population of uniform size small air bubbles in a one-dimensional resonator cavity. This bubbly liquid is both highly nonlinear and dispersive, and has complex attenuation properties [17]. No acoustic perturbations or bubble vibrations are considered at the outset. A pressure source is set at one end $x = 0$ m and a free wall is considered at the other end $x = l_x$. The source excites the bubbly liquid at two driving frequencies: f_1 and f_2 via the following time-dependent function $p_0 g(f_1, f_2, t)$, for which the same amplitude p_0 is used. The cavity is chosen to be resonant at the difference frequency $f_d = f_2 - f_1$, by setting its length at ¾ of the wavelength at f_d in the bubbly liquid, which magnifies the generation of the difference frequency component. The frequencies used in the numerical calculations presented below are chosen to be low enough below the bubble resonance to avoid strong attenuation that would limit the difference frequency generation.

In all the following numerical experiments we choose an initial bubble radius of $R_{0g} = 2.5$ μm that corresponds to a resonance frequency $f_r = 1.35$ MHz, and for their density in water we set a value of $N_g = 5 \times 10^{11}$ m^{-3} (void fraction of 0.0033%). Note that buoyancy is not taken into account in this study and homogeneous bubble distribution is maintained. We set the primary frequencies at $f_1 = 0.7$ MHz and $f_2 = 0.9$ MHz ($f_1/f_r = 0.52$ and $f_2/f_r = 0.66$). Their predicted phase speeds in the bubbly liquid are $c_1 = 1160$ m s^{-1} and $c_2 = 1094$ m s^{-1}, respectively [17]. The difference frequency is $f_d = 200$ kHz ($f_d/f_r = 0.15$), which means that its predicted phase speed in the considered bubbly liquid is $c_d = 1223$ m s^{-1} and the length of the cavity is thus set at $l_x = 4.6$ mm. 3.5 periods at f_1 and 4.5 periods at f_2 correspond to 1 cycle at f_d.

The situation described above suits the possibility of the SnowBl numerical code [18], which solves, from time $t = 0$ s up to $t = l_t$, a differential system coupling the behavior of the acoustic field in the cavity (wave equation written in acoustic pressure $p(x,t)$) to the vibration of the bubbles (Rayleigh–Plesset equation written in bubble-volume variation $v(x,t)$, i.e., the difference between the current volume and the initial bubble volume v_{0g}) by means of a finite-differences algorithm. The coupled system was described in Refs. [9,10,17]:

$$\begin{cases} \frac{\partial^2 v}{\partial t^2} + \delta\omega_r \frac{\partial v}{\partial t} + \omega_r^2 v + \eta p = a v^2 + b\left(2v\frac{\partial^2 v}{\partial t^2} + \left(\frac{\partial v}{\partial t}\right)^2\right), \\ \quad 0 \leqslant x \leqslant l_x, \;\; 0 < t < l_t \\ \frac{\partial^2 p}{\partial x^2} - \frac{1}{c_{0l}^2}\frac{\partial^2 p}{\partial t^2} = -\rho_{0l} N_g \frac{\partial^2 v}{\partial t^2}, \;\; 0 < x < l_x, \;\; 0 < t < l_t \end{cases}, \tag{1}$$

and the auxiliary conditions are

$$\begin{array}{ll} v(x,0) = 0,\; \partial v/\partial t(x,0) = 0 \;\; 0 \leqslant x \leqslant l_x & p(0,t) = p_0 g(f_1, f_2, t) \;\; 0 \leqslant t \leqslant l_t \\ p(x,0) = 0,\; \partial p/\partial t(x,0) = 0 \;\; 0 < x \leqslant l_x & p(l_x,t) = 0 \;\; 0 \leqslant t \leqslant l_t \end{array}. \tag{2}$$

In these equations, $\omega_r = 2\pi f_r$, c_{0l} is the small-amplitude sound speed of the liquid, ρ_{0l} is the equilibrium density of the liquid, $\delta = 4\mu_k/\omega_r R_{0g}^2$ is the viscous damping coefficient of the bubbly liquid, μ_k is the kinematic viscosity of the liquid, $\eta = 4\pi R_{0g}/\rho_{0l}$, $a = (\gamma_g + 1)\omega_r^2/2v_{0g}$, $b = 1/6v_{0g}$, and γ_g is the specific heat ratio of the gas.

The following restrictions are applied to this system: (i) the nonlinear phenomena are due to the bubbles only; (ii) the void fraction is much lower than 1; (iii) the bubbles are of uniform size and spherical; (iv) the bubble pulsations are spherically symmetric; (v) the wavelength in the bubbly liquid is large compared with the bubble radius; (vi) there are no interactions among bubbles; (vii) the surface tension of the bubbles is considered negligible; (viii) the bubbles do not radiate sound themselves; and (ix) the adiabatic gas law holds. The solution gives the acoustic pressure and bubble volume variation fields which are both nonlinear at high amplitudes.

3. Results

Fig. 1 displays the waveform of acoustic pressure at $x = l_x/2$ during the last periods. The curves in Fig. 1 refer to a small amplitude case (linear regime, first point in Fig. 3) and to a high amplitude case (nonlinear regime, last point in Fig. 3). Note that the steady state was reached in all the simulations run and presented here. The comparison of these waveforms shows the existence of distortion when amplitudes are high, which includes a sharpening of wave fronts and an asymmetry of negative and positive acoustic pressures (oscillations traducing the existence of new frequencies), due to the nonlinearity of the medium, whereas the small amplitude signal shows a signal governed by the two driving frequencies only.