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Hybrid phononic crystal plates for lowering and widening acoustic band gaps

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ABSTRACT

We propose hybrid phononic-crystal plates which are composed of periodic stepped pillars and periodic holes to lower and widen acoustic band gaps. The acoustic waves scattered simultaneously by the pillars and holes in a relevant frequency range can generate low and wide acoustic forbidden bands. We introduce an alternative double-sided arrangement of the periodic stepped pillars for an enlarged pillars' head diameter in the hybrid structure and optimize the hole diameter to further lower and widen the acoustic band gaps. The lowering and widening effects are simultaneously achieved by reducing the frequencies of locally resonant pillar modes and prohibiting suitable frequency bands of propagating plate modes.

1. Introduction

The propagation of acoustic and elastic waves in periodic composite materials, sonic crystals, or phononic crystals (PCs) has attracted considerable interest and attention over the past two decades [1–4]. The unique physical properties they produce may lead to the development of applications in low frequency for sound insulation [4–9] and environmental noise control, and in high frequency for filtering and sensing [10–14]. Particularly, microfabricated PCs have recently been adopted in micromechanical resonators and filters for device performance enhancement and novel designs [15–19]. Several studies have investigated different types and configurations of periodic structures for low frequency regimes, including PC and locally resonant sonic crystal (LRSC) [5,7,9,20–27], and have reported on one-, two-, and three-dimensional (1D, 2D, and 3D, respectively) periodic systems by which a complete low-frequency band gap (BG) can be opened, based on either the Bragg scattering or local resonance (LR) mechanism. Other investigations have been conducted that propose systems of 2D phononic plates, allowing the opening of low-frequency BG. Wu et al. [23] and Pennec et al. [24] introduced the concept of pillar resonators on plates. They investigated low-frequency BG creation by both LR and Bragg scattering mechanisms, making use of simple pillars on thin elastic plates. Oudich et al. [25] and Hsu [26] reported on the same concept but making use of composite pillars formed by soft and hard materials as a spring-mass resonator, and stepped pillars, respectively. Largely, these works dealt with the LR mechanism as the low-frequency BG opening was considered. Little attention [28–30] has been paid to investigate the widening of such a BG and more importantly, to how to simultaneously widen and lower the frequency BG, even though the typical locally resonant BG is very narrow.

Two major types of PC plates have been investigated in regard to the BG opening: hole-type [10,31–33] and pillar-type [23–26,34]. The hole-type PC plate permits the Bragg scattering mechanism, exclusively, which can open only a small BG at a frequency directly imposed by the period of the structure, and making use of a large filling fraction (i.e., hole diameter). However, the pillar-type structure can create BGs from both LR and Bragg scattering (usually at a higher frequency compared to LR). Low-frequency BGs, due to the pillar's LR modes, are narrow while Bragg BGs, associated with the periodicity of the structure, is relatively wide, and is difficult to lower towards the LR BG range. Therefore, it is more appropriate to use hole-type plates for high frequency and pillar-type for low frequency. If the PC plate could be modified to increase the LR mechanism, a wider BG could be opened in a lower frequency range than the hole-type structure. In addition, this structure supports simultaneous pillar and plate modes. The

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frequencies of pillar modes can be tuned by changing the material composition or the geometry of pillars, with the potential restriction that pillar diameters are smaller than lattice spacing. The frequencies of the plate modes can be tuned by changing the plate thickness and/or increasing the lattice spacing, which will enlarge the structure. We propose here a combination of the specific advantages of hole-type and pillar-type PC plates to simultaneously widen and lower frequency BGs without enlarging the structure.

In this paper, we present the concept of hybrid behaviors for a 2D PC plate, composed of stepped pillars alternatively arranged, double-sided, of a thin holey plate, which we have called a hybrid PC plate. Using this hybrid structure in comparison with the classical pillar-type with single-sided or coaxial double-sided configurations, we show that a significant widening and lowering of acoustic BG can be obtained and physically explained. We use theoretical considerations and numerical computations to probe the physical behavior of this hybrid PC plate and to compare with conventional pillar-type and hole-type structures.

2. Method

Fig. 1(a–c) shows the unit cells of three pillar-type PC plate structures investigated in this study. The pillar-type PCs have the mechanisms of LR and Bragg scattering, and, thus, may exhibit simultaneous low-frequency and Bragg BGs. An alternative PC plate with stepped pillars, shaped to have a neck of reduced diameter and a large head, allows effective shifting of BG ranges by changing the stepped-pillar geometries, especially the size of the head [26]. A conventional arrangement of the pillars on a thin plate is shown in Fig. 1(a). By increasing the head diameter, the effective mass can be increased to lower the acoustic band frequencies. Fig. 1(b) shows the unit cell of a coaxial double-sided PC plate structure of stepped pillars. The coaxial arrangement may enhance the coupling of the aligned locally resonant pillars for modulating the low-frequency BGs. The head size (i.e., the diameter, d_1) of the stepped pillar in both the single-sided and coaxial double-sided structures must be smaller than their lattice spacing, a. To realize this geometrical constraint in the pillar-type PC plates, we propose a unit cell with alternative double-sided pillars in a PC plate, as shown in Fig. 1(c). The arrangement allows a larger resonant head to vary (widen and lower) the BG ranges. Though the real lattice spacing of the arrangement of the unit cell in Fig. 1(c) is doubled (i.e., $2a$), the dimension doubling is only necessary on the periodic plane. The plate thickness and pillar height are preserved, without losing its geometrical compactness. In the following calculations, the dispersion relations for the alternative arrangement were associated with its real lattice spacing. Correspondingly, Fig. 1(d–f) shows our hybrid PC designs that consist of both periodic stepped pillars and periodic holes to incorporate stronger Bragg scattering into the locally resonant PC structures. The holey plates with spatial period of the holes the same as the lattice spacing, a, of the square lattice, can intrinsically enhance the multiple scattering of acoustic waves in the Bragg regime.

To investigate the acoustic BGs and resonances in the proposed pillar-type and hybrid PC plate structures, calculations of the acoustic dispersion relations were conducted using the finite element method (FEM). Commercial software, COMSOL Multiphysics [35], was adopted to implement the FEM calculation procedures. Unit cells, as shown in Fig. 1, were numerically modeled with periodic boundary conditions according to the Bloch theorem. The base plate has thickness e, the stepped pillars have head diameter and height d_1 and h_1, respectively, and neck diameter and height d_2 and h_2, respectively. The stepped pillars were periodically arranged on one or two sides of the base plate surfaces in a square lattice with lattice spacing a. The structures confine the propagation of elastic waves along the x–y plane. The time–harmonic wave propagation in the elastic structures can be expressed as:

$$\nabla \cdot (\mathbf{c}(\mathbf{r}) : \nabla \mathbf{u}(\mathbf{r})) + \rho(\mathbf{r})\omega^2 \mathbf{u}(\mathbf{r}) = 0, \tag{1}$$

where $\nabla = (\partial/\partial x, \partial/\partial y, \partial/\partial z)$ is the 3D differential operator; $\mathbf{r} = (x, y, z)$ denotes the position vector; $\mathbf{u}(\mathbf{r})$ is the elastic displacement vector; $\omega = 2\pi f$ is the angular frequency, with f being the frequency; $\rho(\mathbf{r})$ is the mass density; and $\mathbf{c}(\mathbf{r})$ is the elastic stiffness tensor. According to the Bloch theorem, the elastic displacement field, $\mathbf{u}(\mathbf{r})$, of any periodic elastic system has the following form:

$$\mathbf{u}(\mathbf{r}) = \mathbf{u}_{\mathbf{k}}(\mathbf{r}) \exp(i\mathbf{k} \cdot \mathbf{r}), \tag{2}$$

where $\mathbf{k} = (k_x, k_y)$ is the Bloch wavevector in the irreducible first Brillouin zone, and $\mathbf{u}_{\mathbf{k}}(\mathbf{r})$ is a periodic vector function having spatial periodicity the same as the periodic elastic system. To transfer the Bloch theorem to the FEM model, the elastic displacement vector in Eq. (2) yields periodic boundary conditions based on the Bloch theorem: On the circumference of a square base plate of a unit cell, the spatial part of the time-harmonic displacement vector is:

$$\mathbf{u}(\mathbf{r} + \mathbf{p}) = \mathbf{u}(\mathbf{r}) \exp(i\mathbf{k} \cdot \mathbf{p}), \tag{3}$$

where $\mathbf{p}$ is the spatial period vector of the PC plate structures, which is related to the lattice symmetry (i.e., square lattice in this study) and the lattice spacing, a. Note that for the alternative double-sided pillars, as shown in Fig. 1(c and f), the enlarged unit-cell method was applied, in which the enlarged period corresponds to $2a$ and

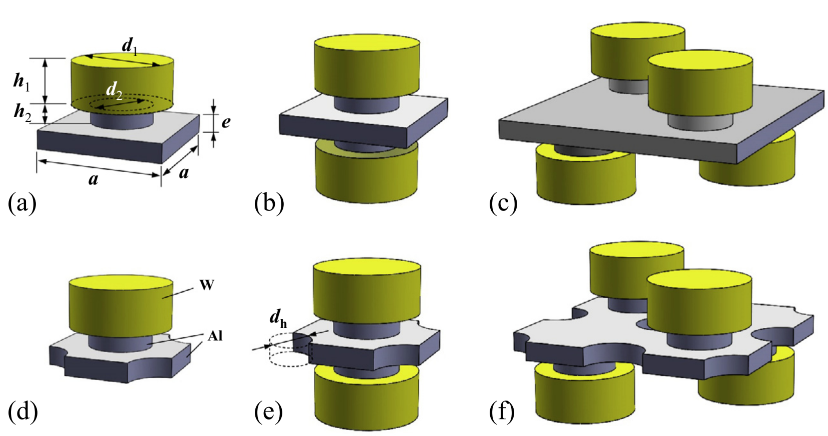

Fig. 1. Unit cells of the square-lattice pillar-type and hybrid PC plates. In all the unit cells, the heads are assumed to be made of W, and the necks and base plates are assumed to be made of Al. The hole diameter in the hybrid PC plates is denoted by d_h.