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Sound field separating on arbitrary surfaces enclosing a sound scatterer based on combined integral equations

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ABSTRACT

To eliminate the limitations of the conventional sound field separation methods which are only applicable to regular surfaces, a sound field separation method based on combined integral equations is proposed to separate sound fields directly in the spatial domain. In virtue of the Helmholtz integral equations for the incident and scattering fields outside a sound scatterer, combined integral equations are derived for sound field separation, which build the quantitative relationship between the sound fields on two arbitrary separation surfaces enclosing the sound scatterer. Through boundary element discretization of the two surfaces, corresponding systems of linear equations are obtained for practical application. Numerical simulations are performed for sound field separation on different shaped surfaces. The influences induced by the aspect ratio of the separation surfaces and the signal noise in the measurement data are also investigated. The separated incident and scattering sound fields agree well with the original corresponding fields described by analytical expressions, which validates the effectiveness and accuracy of the combined integral equations based separation method.

1. Introduction

Sound field separation, which decomposes the total composite sound field to the incident and scattering partial fields on a surface enclosing a sound scatterer, is an important subject in many acoustic applications. For near-field acoustic holography (NAH) of vibrating structures mounted in non-anechoic environment, sound fields propagating inward to the measurement surfaces should be extracted and removed from the total sound field [1,2]. Besides, in contactless micro-manipulation by means of acoustic radiation force, sound field synthesis technology [3] is applied to produce the desired sound field that corresponds with the specific distribution of acoustic radiation force [4,5]. However, obstacles often exist, which make the sound field synthesis domain a non-free space. To guarantee the applicability of the sound field synthesis, sound field separation technique is used to overcome adverse disturbances in the propagation operator caused by the obstacle's scattering field. Other applications necessitating sound field separation include characteristic parameter measurement for acoustic material [6], acoustic target strength characterization in underwater acoustics [7], *etc.*

In conventional sound field separation methods, spatial Fourier transform (SFT) is applied to convert the total sound field from spatial domain to wave-number domain, in which subsequent sound field separating operation is accomplished. Weinreich and Arnold [8] proposed a method to decompose double layer sound fields with spherical harmonic functions. Frisk et al. [9] derived a SFT-based method for measuring the reflection coefficient of the seafloor. Tamura et al. [10,11] developed a method to measure the plane-wave reflection coefficient at oblique incidence. Cheng et al. [12] extended that method, by which the scattering field from a complex shaped object is separated with the incident field in Cartesian and cylindrical coordinates. This method was further applied by Yu et al. [13] to achieve sound field separation in spherical coordinates.

The major advantage of the SFT-based methods mentioned above is their simplicity and efficiency in application. However, due to the inherent characteristics of SFT, these methods can only accomplish sound field separation for three types of regular surfaces: those with planar, cylindrical, and spherical geometries. In many other situations, irregularly shaped separation surfaces are often involved. For example, to obtain near-field information on the scattering field from an irregular sound scatterer, the separation surfaces have to be conformal with the physical frontier of the scatterer and then become irregular accordingly. In addition, to enlarge the applicable region dimension of sound field synthesis,

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the spatial points on which the propagation operator values are measured should be distributed over a surface as close to the obstacle as possible. Therefore, if the obstacle is irregular, sound field separation on irregularly shaped surfaces will be required. Besides, in SFT-based separation process, wave-number spectrum of the sound pressure on the double measurement surfaces should be truncated at the same order. This requirement leads to amplification of the measurement error due to the ill-posedness effect of inverse problems.

Up to now, there have been developed several methods for sound field separation on irregular surfaces. Bi et al. [14,15] proposed a technique based on the equivalent source method. In its application, sound pressure fields are measured and separated on two closely-spaced and parallel surfaces. Based on the fundamental solution of the Helmholtz equation and the wave superposition principle, Fernandez-Grande et al. [16,17] developed a method to deal with sound field separation on non-separable geometries. The common idea of the methods in Ref. [14–17] is to approximate the original sound fields by linear combination of the component fields from the equivalent sources, then solve the weight coefficients by matching the assumed-form solution to the measurement data on the separation surfaces. Since the separating operation is performed directly in the spatial domain instead of the wave-number domain, these techniques are applicable to arbitrary surfaces, and the limitations (such as window effects) due to SFT can be avoided. A difficulty in using these equivalent source based methods is that there exists little theoretical guidance to determine the proper location, order (or type) and number of the equivalent sources, although separation effect is closely related with these characteristics. Based on the Helmholtz integral equation, Langrenne et al. [18,19] developed a technique for sound field separation on general surfaces. For its implementation, normal velocity on the objective separation surface is obtained indirectly by finite difference of sound pressure, which are measured on two adjacent and conformal surfaces. Owing to the ill-posedness of the finite difference operation, small noise in the measurement data might induce large error in the approximated normal velocity [20], and therefore make the separation results depart from the corresponding original fields.

To overcome the limitations of these existing approaches, a sound field separation method is proposed in this study based on combined integral equations. In Section 2, three separation methods in the form of combined integral equations are established theoretically. In Section 3, three corresponding systems of linear equations are obtained through boundary element discretization of the separation surfaces. Numerical simulations on sound field separation under different conditions are carried out in Section 4. Conclusions and suggestions for future research are given in Section 5.

2. Sound field separation based on combined integral equations

In the exterior domain of an arbitrarily shaped sound scatterer, the total sound pressure $p_t(\mathbf{r},t)$ consists of two parts, i.e., the original incident sound pressure $p_{in}(\mathbf{r},t)$ and the scattering sound pressure $p_{sc}(\mathbf{r},t)$. The geometry of interest is depicted in Fig. 1. The domain occupied by the scatterer is denoted by Ω and its physical frontier by S_0. S_1 and S_2 are two arbitrarily shaped surfaces enclosing the scatterer, on which the total sound fields are measured and then separated.

For a time-harmonic $\exp(-i\omega t)$ disturbance of angular frequency ω, the spatial part of sound pressure outside the scatterer satisfies the Helmholtz equation $\Delta p + k^2 p = 0$, in which k is the wave number. When subjected to the Sommerfeld radiation condition and the boundary condition on the scatterer surface, boundary integral equations can be found. These boundary integral equations are often classified as direct or indirect, where direct corresponds to the Helmholtz integral equation and the numerous indirect equations are based on layer potentials.

The Helmholtz integral equation satisfied by the spatial part $p_{in}(\mathbf{r})$ of the incident sound pressure is

$$\alpha \times p_{in}(\mathbf{r}) = \oiint_S \left[p_{in}(\mathbf{r}')\frac{\partial}{\partial n}G(\mathbf{r},\mathbf{r}') - i\rho_0\omega v_n^{in}(\mathbf{r}')G(\mathbf{r},\mathbf{r}')\right] dS(\mathbf{r}'), \tag{1}$$

where the integral surface S is an arbitrary smooth surface enclosing the scatterer, G is the free space Green function, ρ_0 is the equilibrium density of the sound medium, n is the unit interior normal to S, v_n^{in} is the incident normal velocity along the normal direction n [21]. The value of the coefficient α varies as

$$\alpha = \begin{cases} 0 & \text{if } \mathbf{r} \text{ is outside S} \\ \frac{1}{2} & \text{if } \mathbf{r} \text{ is on S} \\ 1 & \text{if } \mathbf{r} \text{ is inside S} \end{cases}.$$

According to Eq. (1) for $\alpha = 0$, the incident sound pressure and normal velocity on S_1 can be applied to compute the incident sound pressure on S_2, i.e.,

$$0 = \oiint_{S_1} \left[p_{in}(\mathbf{r}_1)\frac{\partial}{\partial n}G(\mathbf{r}_2,\mathbf{r}_1) - i\rho_0\omega v_n^{in}(\mathbf{r}_1)G(\mathbf{r}_2,\mathbf{r}_1)\right] dS(\mathbf{r}_1). \tag{2}$$

Similarly, the incident sound pressure and normal velocity on S_2 can also be applied to compute the incident sound pressure on S_1

$$p_{in}(\mathbf{r}_1) = \oiint_{S_2} \left[p_{in}(\mathbf{r}_2)\frac{\partial}{\partial n}G(\mathbf{r}_1,\mathbf{r}_2) - i\rho_0\omega v_n^{in}(\mathbf{r}_2)G(\mathbf{r}_1,\mathbf{r}_2)\right] dS(\mathbf{r}_2). \tag{3}$$

The Helmholtz integral equation satisfied by the spatial part $p_t(\mathbf{r})$ of the total sound pressure is

$$\alpha \times p_t(\mathbf{r}) = p_{in}(\mathbf{r}) + \oiint_S \left[p_t(\mathbf{r}')\frac{\partial}{\partial n}G(\mathbf{r},\mathbf{r}') - i\rho_0\omega v_n^{t}(\mathbf{r}')G(\mathbf{r},\mathbf{r}')\right] dS(\mathbf{r}'), \tag{4}$$

where v_n^t is the total normal velocity along the normal direction n [21]. The coefficient

$$\alpha = \begin{cases} 1 & \text{if } \mathbf{r} \text{ is outside S} \\ \frac{1}{2} & \text{if } \mathbf{r} \text{ is on S} \\ 0 & \text{if } \mathbf{r} \text{ is inside S} \end{cases}.$$

In Ref. [18,19], the total sound pressure $p_t(\mathbf{r})$ and the total normal velocity $v_n^t(\mathbf{r})$ on S are both known. Hence the incident sound pressure $p_{in}(\mathbf{r})$ on S can be computed based on Eq. (4), while the scattering sound field $p_{sc}(\mathbf{r})$ on S equals $p_t(\mathbf{r}) - p_{in}(\mathbf{r})$. In this study, we consider the application conditions where only one type of sound field data is known by measurement, either the total sound pressure or the total normal velocity.

When the field point $\mathbf{r}$ is outside the integral surface S, Eq. (4) could be written as

$$\begin{aligned} p_t(\mathbf{r}) &= p_{in}(\mathbf{r}) + \oiint_S \left[p_{in}(\mathbf{r}')\frac{\partial}{\partial n}G(\mathbf{r},\mathbf{r}') - i\rho_0\omega v_n^{in}(\mathbf{r}')G(\mathbf{r},\mathbf{r}')\right] dS(\mathbf{r}') \\ &\quad + \oiint_S \left[p_{sc}(\mathbf{r}')\frac{\partial}{\partial n}G(\mathbf{r},\mathbf{r}') - i\rho_0\omega v_n^{sc}(\mathbf{r}')G(\mathbf{r},\mathbf{r}')\right] dS(\mathbf{r}') \\ &= p_{in}(\mathbf{r}) + \oiint_S \left[p_{sc}(\mathbf{r}')\frac{\partial}{\partial n}G(\mathbf{r},\mathbf{r}') - i\rho_0\omega v_n^{sc}(\mathbf{r}')G(\mathbf{r},\mathbf{r}')\right] dS(\mathbf{r}'). \end{aligned}$$

Since $p_t(\mathbf{r}) = p_{in}(\mathbf{r}) + p_{sc}(\mathbf{r})$, it can be derived from the above formula that in the domain outside S,

$$p_{sc}(\mathbf{r}) = \oiint_S \left[p_{sc}(\mathbf{r}')\frac{\partial}{\partial n}G(\mathbf{r},\mathbf{r}') - i\rho_0\omega v_n^{sc}(\mathbf{r}')G(\mathbf{r},\mathbf{r}')\right] dS(\mathbf{r}'). \tag{5}$$