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Modeling nonlinearities of ultrasonic waves for fatigue damage characterization: Theory, simulation, and experimental validation

Q1 Ming Hong [a,b], Zhongqing Su [a,b,*], Qiang Wang [c], Li Cheng [a,b], Xinlin Qing [d]

[a] The Hong Kong Polytechnic University, Shenzhen Research Institute, Shenzhen 518057, PR China
[b] Department of Mechanical Engineering, The Hong Kong Polytechnic University, Kowloon, Hong Kong
[c] College of Automation, Nanjing University of Posts and Telecommunications, Nanjing, PR China
[d] Division of Aviation Health and Safety Management, Beijing Aeronautical Science and Technology Research Institute COMAC, Beijing 100083, PR China

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A B S T R A C T

A dedicated modeling technique for comprehending nonlinear characteristics of ultrasonic waves traversing in a fatigued medium was developed, based on a retrofitted constitutive relation of the medium by considering the nonlinearities originated from material, fatigue damage, as well as the "breathing" motion of fatigue cracks. Piezoelectric wafers, for exciting and acquiring ultrasonic waves, were integrated in the model. The extracted nonlinearities were calibrated by virtue of an acoustic nonlinearity parameter. The modeling technique was validated experimentally, and the results showed satisfactory consistency in between, both revealing: the developed modeling approach is able to faithfully simulate fatigue crack-incurred nonlinearities manifested in ultrasonic waves; a cumulative growth of the acoustic nonlinearity parameter with increasing wave propagation distance exists; such a parameter acquired via a sensing path is nonlinearly related to the offset distance from the fatigue crack to that sensing path; and neither the incidence angle of the probing wave nor the length of the sensing path impacts on the parameter significantly. This study has yielded a quantitative characterization strategy for fatigue cracks using embeddable piezoelectric sensor networks, facilitating deployment of structural health monitoring which is capable of identifying small-scale damage at an embryo stage and surveilling its growth continuously.

1. Introduction

Effectual damage evaluation and continuous health monitoring are conducive to reliable service of engineering structures, and the risk of structural failure can accordingly be minimized. Taking advantage of appealing features including high sensitivity to structural damage, omnidirectional dissemination, fast propagation, and strong penetration through thickness, ultrasonic waves have been a subject of intense scrutiny over the years, with demonstrated compromise between conventional non-destructive evaluation (NDE) and emerging structural health monitoring (SHM) [1–7]. Predominantly, deployment of this group of techniques is often based on exploring changes in the linear wave scattering upon the interaction of incident probing waves with structural damage. These changes can be manifested in acquired ultrasonic wave signals, typified as delay in time-of-flight, wave attenuation, and mode conversion. These signal features, for example the delay in time-of-flight, show, to some extent, linear correlation with damage parameters such as the location, and are therefore referred to as *linear features*.

However, it is a corollary that linear features-based detection is fairly limited to evaluating damage with a size on the same order of the magnitude of the probing wavelength [8], presenting inefficiency in perceiving fatigue damage which often initiates at an unperceivable level much smaller than the probing wavelength. This is because the damage of small dimension is not anticipated to induce evident changes in linear features to be extracted from ultrasonic waves [9]. This situation has posed immediate urgency and entailed imperative needs for exploring other wave signal features that can be prominently modulated by small-scale damage, so as to endow the ultrasonic inspection with a capability of scrutinizing damage small in dimension and fatigue cracks in particular.

Nonlinear ultrasonic interrogation has emerged under such a demand. More specifically, instead of extracting and canvassing linear signal properties, the nonlinear ultrasonic inspection attempts to quantify the nonlinear distortion of probing waves due to the damage, for instance the generation of higher-order harmonics. Such a detection philosophy has ushered a new avenue of using ultrasonic waves to predict fatigue damage at an embryo stage prior to the formation of gross damage detectable by linear techniques.

Q2 * Corresponding author at: Department of Mechanical Engineering, The Hong Kong Polytechnic University, Kowloon, Hong Kong. Tel.: +852 2766 7818; fax: +852 2365 4703.
E-mail address: MMSU@polyu.edu.hk (Z. Su).

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There has been a rich body of literature on the use of nonlinear features of ultrasonic waves [9–16]. Most of existing research strength has been in a nature of experimental observation on possible changes in nonlinear properties of probing waves, primarily Lamb waves (the modality of elastic waves in thin plate- or shell-like structures) [10–13]. There are rather limited studies devoted to the analytical investigation or numerical simulation of nonlinear ultrasonic waves propagating in fatigued media. Among representative numerical methods to simulate nonlinearities of media and/or damage are Local Interaction Simulation Approach (LISA) [14], finite element method (FEM) [15], and Galerkin FEM [16].

In general, a paramount challenge in analytically or numerically modeling nonlinear ultrasonic waves in a fatigued medium is the comprehensive inclusion of all possible sources of nonlinearities from both the medium itself and the damage, as well as the interpretation on the modulation mechanism of fatigue damage on ultrasonic waves. Aimed at a systematic comprehension of the nonlinear natures of ultrasonic waves in a medium bearing fatigue damage, this study is dedicated to the establishment of a modeling technique — supplemented with experimental validation — that is capable of producing and interpreting nonlinearities in ultrasonic waves. Instead of using bulky wedge probes that are commonly adopted in prevailing nonlinear ultrasonic interrogation, miniaturized piezoelectric wafer sensors, which can be flexibly networked and permanently attached to a structure under inspection, are utilized, benefiting extension of the approach to embeddable SHM.

This paper is organized as follows: Section 2 discusses the origins of nonlinearities in an elastic medium, serving as the cornerstone of the study, residing on which the modeling technique for an ideally intact medium and its fatigued counterpart is developed. An acoustic nonlinearity parameter is established to quantitatively calibrate the captured nonlinearity. By integrating identified sources of nonlinearities, Section 3 models the nonlinear properties manifested in ultrasonic waves traversing in a metallic medium featuring introduced nonlinearities. Section 4 embraces implementation of the modeling through finite element (FE) simulation, and signal processing for extracting nonlinearities from ultrasonic wave signals. Case studies using the developed modeling technique are presented in Section 5, investigating the dependence of the acoustic nonlinearity parameter on wave propagation distance, on sensing path offset from a fatigue crack, and on wave incidence angle and propagation distance. Finally, Section 6 renders concluding remarks.

2. Modeling nonlinearities in elastic medium

Consider an isotropic homogeneous solid with purely elastic behavior, the nonlinearities of the medium that may contribute to nonlinear distortion of its guided ultrasonic waves can originate from different sources, including mainly the material, the damage-driven plasticity, the loading conditions, to name a few.

2.1. Intact state

When the medium is in an ideally intact state (no fatigue damage existent), two nonlinearity sources are accountable: the inherent material nonlinearity and the geometric nonlinearity, with the former from the intrinsic nonlinear elasticity of the medium (*viz.*, the elasticity of lattices). Usually, lattice vibrations in a metallic medium are assumed to obey simple harmonic motion and the material is assumed to be pristine (*i.e.*, no precipitates or vacancies). This assumption is largely applicable for engineering applications in the domain of linear elasticity. However, in reality there is always lattice anharmonicity (referring to the crystal vibrations that do not follow the simple harmonic motion), and/or there are precipitates and vacancies in the material. These nonlinearity effects, though trivial, can be manifested by ultrasonic waves propagated in such a medium.

In the domain of nonlinear elasticity, the three-dimensional stress–strain relation for the above solid medium can be depicted, with a second-order approximation, as follows [17]

$$\sigma_{ij} = (C_{ijkl} + 1/2M_{ijklmn}\varepsilon_{mn})\varepsilon_{kl}, \tag{1}$$

where σ_{ij} is the stress tensor; ε_{mn} and ε_{kl} are the strain tensors; C_{ijkl} and such in its form in the succeeding equations are the second-order elastic (SOE) tensors defined with Lamé parameters λ_L and μ; M_{ijklmn} is a tensor associated with the material and geometric nonlinearities. If the second term in the parenthesis, $1/2M_{ijklmn}$, is neglected, Eq. (1) reverts to the three-dimensional Hooke's Law of linear elasticity.

In the meantime, the geometric nonlinearity is closely related to the material nonlinearity. Generally, wave motion equations are written in Eulerian (special) coordinates, while nonlinear elasticity in solids is formulated in Lagrangian (material) coordinates. For linear elasticity, these two coordinate systems do not differ from each other; nevertheless, given the material nonlinearity taken into account, a descriptive difference emerges, starting from the second-order term of any physical quantity involved [18]. In simpler words, geometric nonlinearity is induced mainly due to the mathematic transform between two coordinate systems. Hence, tensor M_{ijklmn} in Eq. (1) addresses both the material and geometric nonlinearities simultaneously, which can be expressed in terms of the notation by Landau and Lifshitz [19] as follows

$$M_{ijklmn} = C_{ijklmn} + C_{ijln}\delta_{km} + C_{jnkl}\delta_{im} + C_{jlmn}\delta_{ik} \tag{2}$$

where

$$C_{ijklmn} = \frac{1}{2}\mathcal{A}(\delta_{ik}I_{jlmn} + \delta_{il}I_{jkmn} + \delta_{jk}I_{ilmn} + \delta_{jl}I_{ikmn}) + 2\mathcal{B}(\delta_{ij}I_{klmn} + \delta_{kl}I_{mnij} + \delta_{mn}I_{ijkl}) + 2\mathcal{C}\delta_{ij}\delta_{kl}\delta_{mn}. \tag{3}$$

In Eqs. (2) and (3), δ_{km} and such in its form with different index orders are the Kronecker deltas; I_{jlmn} and such in its form are the fourth-order identity tensors. C_{ijklmn} is the third-order elastic (TOE) tensor describing the material nonlinearity, and the last three terms in Eq. (2) all together address the geometric nonlinearity. As shown in Eq. (3), C_{ijklmn} is determined by three TOE constants $\mathcal{A}$, $\mathcal{B}$ and $\mathcal{C}$, which can be regarded as the inherent properties of the material, to be measured experimentally [20,21]. C_{ijklmn} can further be expressed explicitly with Voigt notation in terms of the three TOE constants, as

$$\begin{cases} c_{111} = 2\mathcal{A} + 6\mathcal{B} + 2\mathcal{C} \\ c_{112} = 2\mathcal{B} + 2\mathcal{C} \\ c_{123} = 2\mathcal{C} \\ c_{144} = 1/2\mathcal{A} + \mathcal{B} \\ c_{155} = \mathcal{B} \\ c_{456} = 1/4\mathcal{A}, \end{cases} \tag{4}$$

where $c_{IJK} = C_{ijklmn}$ ($I, J, K \in \{1, 2, \ldots, 6\}$). For example, the cases that $I = 1, 2, \ldots, 6$ are corresponding to those when ij = 11, 22, 33, 12, 23, 31, respectively, and any other scalar components of C_{ijklmn} fall into the six cases defined by Eq. (4).

For generality, first consider a one-dimensional medium, such as a rod, which can be governed by the one-dimensional nonlinear stress–strain equation as follows:

$$\sigma = (E + E_2\varepsilon)\varepsilon, \tag{5}$$

where σ, ε, E, and E_2 are the stress, strain, and the first- and second-order Young's moduli of the medium, respectively. E reflects the lin-

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