Ultrasonics 54 (2014) 888–893

Contents lists available at ScienceDirect

Ultrasonics

journal homepage: www.elsevier.com/locate/ultras

Analytical one-dimensional model for laser-induced ultrasound in planar optically absorbing layer

Erika Svanström *, Tomas Linder, Torbjörn Löfqvist

EISLAB, Department of Computer Science and Electrical Engineering, Luleå University of Technology, SE-971 87 Luleå, Sweden

A R T I C L E I N F O

Article history:
Received 9 May 2013
Received in revised form 28 October 2013
Accepted 29 October 2013
Available online 7 November 2013

Keywords:
Photoacoustic
Thermoelastic
Optical absorption
Thin film
Polymer

A B S T R A C T

Ultrasound generated by means of laser-based photoacoustic principles are in common use today and applications can be found both in biomedical diagnostics, non-destructive testing and materials characterisation. For certain measurement applications it could be beneficial to shape the generated ultrasound regarding spectral properties and temporal profile. To address this, we studied the generation and propagation of laser-induced ultrasound in a planar, layered structure. We derived an analytical expression for the induced pressure wave, including different physical and optical properties of each layer. A Laplace transform approach was employed in analytically solving the resulting set of photoacoustic wave equations. The results correspond to simulations and were compared to experimental results. To enable the comparison between recorded voltage from the experiments and the calculated pressure we employed a system identification procedure based on physical properties of the ultrasonic transducer to convert the calculated acoustic pressure to voltages. We found reasonable agreement between experimentally obtained voltages and the voltages determined from the calculated acoustic pressure, for the samples studied. The system identification procedure was found to be unstable, however, possibly from violations of material isotropy assumptions by film adhesives and coatings in the experiment. The presented analytical model can serve as a basis when addressing the inverse problem of shaping an acoustic pulse from absorption of a laser pulse in a planar layered structure of elastic materials.

1. Introduction

The use of laser based photoacoustic principles in generation of ultrasound is an efficient way to generate ultrasound pulses of high frequencies and large bandwidths. These properties has been utilised foremost in biomedical imaging and diagnosis [1] but also in e.g. materials characterisation [2], or non-destructive testing [3] or other engineering applications [4]. The technique rests on absorption of an incident laser light pulse and energy conversion through a thermoelastic process resulting in ultrasonic waves [5]. An objective in earlier studies has been to create a high conversion efficiency situation, i.e. maximising the ultrasound pulse energy by optimising the light absorption and energy conversion in the light absorbing layer. This is commonly achieved by opaque light absorbing films or structures as e.g. metal thin films [6], light absorbing polymer thin films [7,8], optically absorbing gold nano structures [9,10], or thin layers of carbon nanotube composites [11].

However, in some applications a desirable feature would be to use a semi-transparent layer that partly absorbs and partly transmits the incoming laser radiation. This could be useful in applications where a traditional, pulse-echo technique is used in conjunction with e.g. photoacoustic tomography. In this case the optical absorption is only partial and the possibility appears to construct the absorbent such that the resulting ultrasound is influenced by the physical layout and material properties of the absorbent. Planar, layered absorbents has been studied earlier within the field of photoacoustic spectroscopy. Sun et al. [12] modelled generation of ultrasonic waves from a photoacoustic source consisting of planar, alternating layers of weakly light-absorbing solid and transparent fluid layers. An amplitude-modulated laser generated acoustic waves at specific resonance frequencies by means of acoustic interference in the layers. In order to shape the resulting ultrasonic pulse, light absorbing or transparent layers may be interleaved with layers having different optical absorption coefficients, thickness's and elastic properties. The stack will then operate basically as a acoustic filter. Hu et al. [13] modelled layers of arbitrary physical properties and calculated the photoacoustic response to sinusoidally modulated heating. A transfer matrix method for calculation of the thermoelastic response of multilayered samples exposed to modulated laser heating was presented by Bozoki et al. [14].

As a base for ultrasound pulse shaping by a pulsed laser, in present work we model, from fundamental principles and for one

* Corresponding author. Tel.: +46 920 493335.
E-mail address: erika.svanstrom@ltu.se (E. Svanström).

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http://dx.doi.org/10.1016/j.ultras.2013.10.019

light pulse, the transient acoustical pulse generation and transmission in a light-absorbing layer. This layer is consisting of a stack of planar isotropic films, and is surrounded by two material layers that can have different physical properties. A Laplace transform approach is used in solving the set of linear, one-dimensional wave equations describing the transient wave propagation in the three-layer structure. The analytical solutions for pressures are compared to simulations of pressures as well as to experimental transducer voltage values. To facilitate the latter comparison, a system identification process of tranducer characteristics based on physical principles, is employed for estimations of the pressure to voltage transition.

2. Analytical modelling

We will analytically express laser-induced ultrasound as acoustic pressure $p = p_i(x,t)$ in position x at time t in a material layer i. The analysis is based on a model presented by Shan et al. [15], of a light-absorbing layer within a fluid. Present work is an extension that enables the properties of the materials on each of the two sides of the light-absorbing layer to be different. In the presented study we only consider one space dimension of the wave propagation problem. It is assumed, for the photoacoustic 1D wave equation that the material in each layer has isotropic properties and is linearly elastic. We assume that the laser pulse width τ_{pulse} is very short, so that the heat conduction is negligible during τ_{pulse} for thermal confinement, and also so that time τ_{stress} of pressure propagation at sound speed c across a characteristic dimension d_c of the heated region as $\tau_{stress} = d_c/c$ [16], fulfils the condition $\tau_{pulse} \ll \tau_{stress}$ for stress confinement. Further, acoustic attenuation is neglected in each layer.

To model the generation and propagation of laser-induced pressure waves, a photoacoustic wave equation is set up for each material layer i. For descriptions of coming denotations, see Table 2.1. The index i numbering of the layers is illustrated in Fig. 2.1, where the index counting starts from the layer closest to the laser source. The boundaries around the light absorbing layer 2 are $x = b_{1,2}$ and $x = b_{2,3}$, where $0 \leqslant b_{1,2} < b_{2,3}$. The 1D photoacoustic wave equation [16] in layer $i = 1, 2, 3$ for pressure $p_i(x,t)$ is

$$\frac{\partial^2 p_i(x,t)}{\partial x^2} - \frac{1}{c_i^2}\frac{\partial^2 p_i(x,t)}{\partial t^2} = -\frac{\beta_i}{C_{pi}}\frac{\partial H_i(x,t)}{\partial t}, \tag{2.1}$$

were the right hand side source term holds the heating function

$$H_1 \equiv 0, \quad H_2(x,t) = \mu_{a2} e^{-\mu_{a2}x} E_{02}\delta(t), \quad H_3 \equiv 0, \tag{2.2}$$

with light-absorption in layer $i = 2$. In stress confinement, laser-pulse width is approximated by a delta function $\delta(t)$.

Initial conditions are

$$p_i(x, 0_-) = 0 \tag{2.3}$$

$$\frac{\partial}{\partial t} p_i(x, 0_-) = 0 \tag{2.4}$$

$$H_2(x, 0_-) = 0, \tag{2.5}$$

Table 2.1
Denotations.

Denotation	Description
β_i	isobaric volume expansion coefficient
C_{pi}	isobaric heat capacity per unit mass
μ_{ai}	optical absorption coefficient
E_{0i}	energy density at layer leftmost surface
$\theta(t)$	Heaviside step function
ρ_i	Density
h_i	Thickness of layer i, $h_i = b_{i,i+1} - b_{i-1,i}$

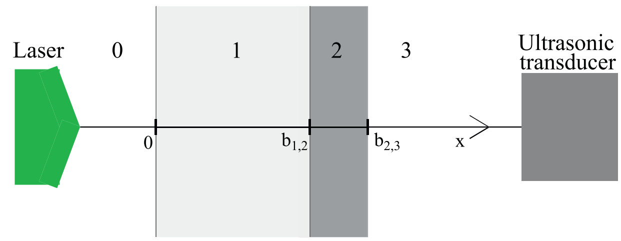

Fig. 2.1. Illustration of material layers $i = 0, 1, 2, 3$. The light-absorbing layer 2 has boundaries ($x = b_{1,2}$ and $x = b_{2,3}$. Heating from a laser pulse induces ultrasound in layer 2, which is recorded by an ultrasound transducer placed in material 3.

and boundary conditions

$$p_i(b_{i,i+1}, t) = p_{i+1}(b_{i,i+1}, t), \tag{2.6}$$

$$-\frac{1}{\rho_i}\frac{d}{dx}p_i(b_{i,i+1}, t) = -\frac{1}{\rho_{i+1}}\frac{d}{dx}p_{i+1}(b_{i,i+1}, t), \tag{2.7}$$

are continuity in pressure and interface acceleration across each interface $x = b_{i,i+1}$ between layer i and $i + 1$.

The set of i photoacoustic wave Eq. (2.1), including boundary and initial conditions in Eqs. (2.3)–(2.7), are Laplace transformed

$$L\{p_i(x,t)\}(t \to s) = y_i(x,s), \tag{2.8}$$

and solved in the transform plane. General solutions with coefficients $C_{iF}(s)$ and $C_{iB}(s)$ for, in x-direction, forward and backward propagating waves in layer i respectively, are for layer $i = 1, 2, 3$

$$y_1(x,s) = C_{1B}(s)e^{\frac{sx}{c_1}} \tag{2.9}$$

$$y_2(x,s) = C_{2F}(s)e^{-\frac{sx}{c_2}} + C_{2B}(s)e^{\frac{sx}{c_2}} + \frac{F_2 s e^{-\mu_{a2}x}}{s^2 - c_2^2\mu_{a2}^2} \tag{2.10}$$

$$y_3(x,s) = C_{3F}(s)e^{-\frac{sx}{c_3}}, \tag{2.11}$$

where we used that the indirect condition on bounded pressure in $x = \pm\infty$ gives $C_{1F} \equiv 0$ and $C_{3B} \equiv 0$. Material parameters are assembled into

$$F_2 = \frac{\mu_{a2}\beta_2 E_{02} c_2^2}{C_{p2}}. \tag{2.12}$$

The coefficients $C_{iF}(s)$ and $C_{iB}(s)$ are obtained from an equation system of the two boundary conditions for each of the two boundaries. Of interest for comparison with measurements is pressure wave in layer 3. Corresponding coefficient $C_{3F}(s)$ is

$$C_{3F}(s) = e^{\frac{sb_{2,3}}{c_3}}\left(C_{2F}(s)e^{-\frac{sb_{2,3}}{c_2}} + C_{2B}(s)e^{\frac{sb_{2,3}}{c_2}} + \frac{sF_2 e^{-\mu_{a2}b_{2,3}}}{s^2 - c_2^2\mu_{a2}^2}\right), \tag{2.13}$$

with

$$C_{2FD}(s) = \frac{C_{2F}(s)}{D}, C_{2BD}(s) = \frac{C_{2B}(s)}{D}, \tag{2.14}$$

$$D = e^{\frac{sb_{1,2}}{c_2}}\left(\frac{\rho_1 c_1}{\rho_2 c_2} - 1\right) - e^{\frac{s\left(2\cdot b_{2,3} - b_{1,2}\right)}{c_2}}\frac{\left(\frac{\rho_1 c_1}{\rho_2 c_2} + 1\right)\left(\frac{\rho_3 c_3}{\rho_2 c_2} + 1\right)}{\left(\frac{\rho_3 c_3}{\rho_2 c_2} - 1\right)}, \tag{2.15}$$

$$C_{2FD}(s) = \left(\frac{\left(\frac{\rho_1 c_1}{\rho_2 c_2} - 1\right)}{\left(\frac{\rho_3 c_3}{\rho_2 c_2} - 1\right)} F_2 e^{-\mu_{a2} b_{2,3}}\right)\left(e^{\frac{s\left(b_{1,2} + b_{2,3}\right)}{c_2}}\frac{-\frac{\rho_3 c_3}{\rho_2 c_2}\mu_{a2}c_2 + s}{s^2 - c_2^2\mu_{a2}^2}\right) + \left(\frac{\left(\frac{\rho_3 c_3}{\rho_2 c_2} + 1\right)}{\left(\frac{\rho_3 c_3}{\rho_2 c_2} - 1\right)} F_2 e^{-\mu_{a2} b_{1,2}}\right)\left(e^{\frac{s\cdot 2b_{2,3}}{c_2}}, \frac{s + \frac{\rho_1 c_1}{\rho_2 c_2}\mu_{a2}c_2}{s^2 - c_2^2\mu_{a2}^2}\right) \tag{2.16}$$