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Short Communication

Computation of leaky guided waves dispersion spectrum using vibroacoustic analyses and the Matrix Pencil Method: A validation study for immersed rectangular waveguides

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ABSTRACT

The paper aims at validating a recently proposed Semi Analytical Finite Element (SAFE) formulation coupled with a 2.5D Boundary Element Method (2.5D BEM) for the extraction of dispersion data in immersed waveguides of generic cross-section. To this end, three-dimensional vibroacoustic analyses are carried out on two waveguides of square and rectangular cross-section immersed in water using the commercial Finite Element software Abaqus/Explicit. Real wavenumber and attenuation dispersive data are extracted by means of a modified Matrix Pencil Method. It is demonstrated that the results obtained using the two techniques are in very good agreement.

1. Introduction

The knowledge of leaky guided waves dispersion spectrum is of great importance in the NDE testing of immersed and embedded waveguides [1,2].

The dispersion properties of open plate-like and cylindrical waveguides have been addressed using analytical methods in a number of publications [1–7], while only few works focus on waveguides of generic cross-section [8].

Numerical methods that need a discretization of the unbounded external medium include the Semi-Analytical Finite Element (SAFE) method coupled with non-reflective boundary conditions [9], absorbing regions [10–12] and Perfectly Matched Layers (PMLs) [13,14].

In contrast to the numerical methods cited above, the Scaled Boundary Finite Element Method (SBFEM) coupled with dashpot boundary conditions [15] and the SAFE method proposed in [16] do not require a mesh of the surrounding medium (i.e. no spurious solutions are found). In addition to this characteristic, the coupled SAFE–2.5D BEM method [17,18] also have the unique capability to accurately describe irregular domains of infinite extension through exact radiation conditions.

The aim of the present work is to validate the complete dispersion spectra obtained via the SAFE–2.5D BEM method by using the commercial software Abaqus/Explicit [19] and the modified Matrix Pencil Method (MPM) proposed in [20]. To this end, three-dimensional vibroacoustic analyses are performed on square and rectangular waveguides, for which dispersion data are then extracted by means of the MPM for leaky and trapped guided wave modes.

2. Modelling and simulations

Two waveguides of different geometry are considered in the following: a steel bar of 10×10 mm square cross-section and an aluminium bar of 12.7×3.175 mm rectangular cross-section. The bars are immersed in water and have invariant geometric and mechanical properties along the z-axis (propagation axis), while the materials are assumed perfectly elastic. Therefore, wave attenuation is only due to leakage phenomena.

The motion is assumed in the harmonic form $e^{\mathrm{i}(\kappa_z z - \omega t)}$, being $\mathrm{i} = \sqrt{-1}$, κ_z the complex axial wavenumber, ω the circular frequency and t the time, whereas the dispersive parameters are identified as the number of spatial harmonics per unit of length, $\mathrm{Re}(\kappa_z)$, and the amplitude decay per unit of distance traveled (attenuation), $\mathrm{Im}(\kappa_z)$.

Using the SAFE–2.5D BEM method, only the cross-section of the bars needs to be modelled, as shown in Fig. 1(a). In particular, a

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mesh of 6×6 quadrilateral eight-nodes SAFE elements and 36 monodimensional three-nodes BEM elements has been assumed for the square bar, while the rectangular bar has been discretized using a mesh of 6×2 quadrilateral eight-nodes SAFE elements and 16 monodimensional three-nodes BEM elements. The material constants used in the analyses are listed in Table 1.

In Abaqus/Explicit [19], the two open waveguides have been modelled without making use of absorbing regions and by assuming the extension of the waveguide and the fluid equal to $L = 1000$ mm and $d = 800$ mm, respectively (see Fig. 1(b)). Reflections from the free ends and the outer boundary of the fluid domain have been avoided by limiting appropriately the length of time signals.

To reduce computational efforts, the double symmetry properties of the rectangular cross-section has been exploited by considering a quarter of bar as shown in Fig. 1(b) and applying appropriate boundary conditions in relation to the different properties of the wave motion with respect to the four axes of symmetry. With reference to Fig. 1(c–f), on a generic plane $\mathcal{S}$ containing an axis of symmetry s and the z-axis a symmetric boundary condition requires any displacement normal to $\mathcal{S}$ to be zero. Similarly, the boundary conditions at any plane $\mathcal{A}$ containing an axis of antisymmetry a and the z-axis are imposed by letting vanish the displacement components parallel to $\mathcal{A}$ on the solid part as well as the acoustic pressure on the fluid part.

The finite element meshes used in the analyses are composed of tetrahedral and hexahedral elements with linear shape functions. The maximum element size ranges in the xy-plane from a minimum of 0.4 mm to a maximum of 2.0 mm, while a fixed side length of 1.0 mm has been assumed in the z-direction. By adopting a minimum of 10 elements with linear shape functions per wavelength [21], the FEM mesh described above leads to dispersion curves that are accurate up to 147.8 kHz.

In all the analyses, a half-cycle burst has been applied on a fixed point of the free end cross-section of the solid waveguide. For each case of Fig. 1, a space–time array of accelerations $a(z,t)$ has been recorded at $M = 150$ locations equally spaced with $\Delta z = 1.0$ mm (z = 40–190 mm) and N = 3200 time samples with Δt = 1.0e−7 s.

Table 1
Materials constants.

Material	Mass density ρ (kg/m^3)	L-wave speed c_L, c_f (m/s)	S-wave speed c_S (m/s)
Steel	7932	5960	3260
Aluminium	2700	6149	3097
Water	998.2	1478	–

3. Dispersion data extraction via the Matrix Pencil Method

The complex axial wavenumbers of an expected number q of guided wave modes in $a(z,t)$ are extracted at various frequencies by using the modified Matrix Pencil Method proposed in [20].

In the first step of the method, the following Hankel matrix is formed

$$\mathbf{X} = \begin{bmatrix} A(z_1,\omega_0) & A(z_2,\omega_0) & \cdot & A(z_{p+1},\omega_0) \\ A(z_2,\omega_0) & A(z_3,\omega_0) & \cdot & A(z_{p+2},\omega_0) \\ \cdot & \cdot & \cdot & \cdot \\ A(z_{M-p},\omega_0) & A(z_{M-p+1},\omega_0) & \cdot & A(z_M,\omega_0), \end{bmatrix}, \qquad (1)$$

where $z_m = m\Delta z$ $(m = 1,\dots,M)$ is the mth spatial receiver, ω_0 is a fixed frequency at which dispersion data are sought, p is the pencil parameter satisfying the condition $q \leqslant p \leqslant M - q$ and

$$A(z_m,\omega_0) = \underbrace{\sum\nolimits_{n=1}^{N} a(z_m,t_n)e^{\mathrm{i}\omega_0 t_n}}_{\mathrm{DTFT}(a(z_m,t))} = \sum_{j=1}^{p} A(\kappa_z^{(j)},\omega_0)e^{-\mathrm{i}\kappa_z^{(j)}(\omega_0)z_m}, \qquad (2)$$

is a sequence of p complex exponentials that approximates the spatial signal at ω_0, being $t_n = n\Delta t$ $(n = 1,\dots,N)$ and DTFT$(\cdot)$

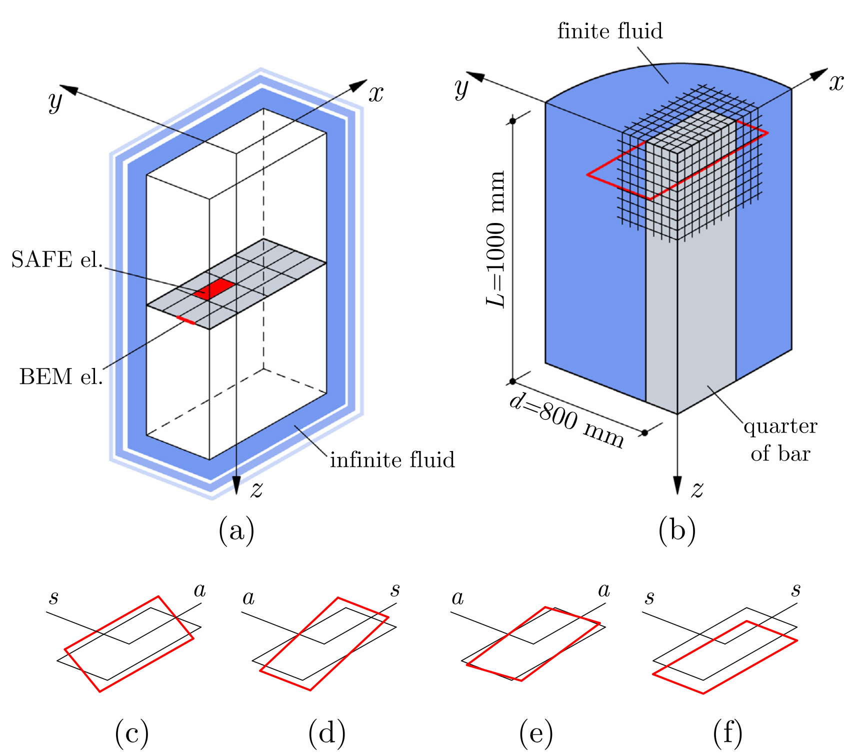

Fig. 1. SAFE–2.5D BEM model (a), Abaqus model (b) and boundary conditions for: slow flexural modes (c), fast flexural modes (d), torsional modes (e) and longitudinal modes (f). Axes of symmetry are indicated with s, while axes of antisymmetry are denoted with a.